

Motivation

Global solution  
theory

Global solution  
at VIE AC

Global solution  
datum definition

Conclusion &  
Outlook



# Current status and future perspectives of VLBI global solutions

EGU General Assembly 2023, 23rd – 28th April 2023, Vienna, Austria

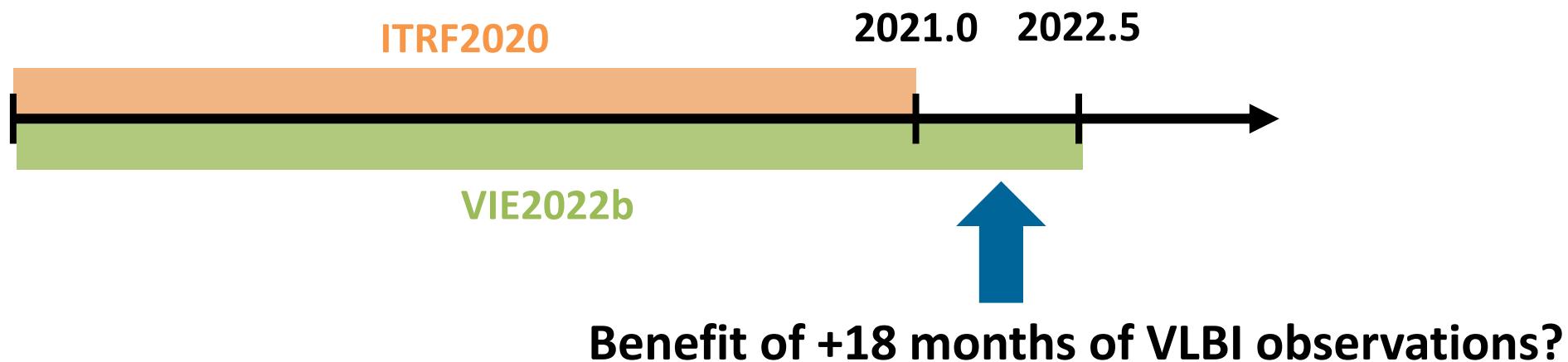
Lisa Kern • Hana Krásná • Johannes Böhm • Matthias Madzak

TU Wien, Austria



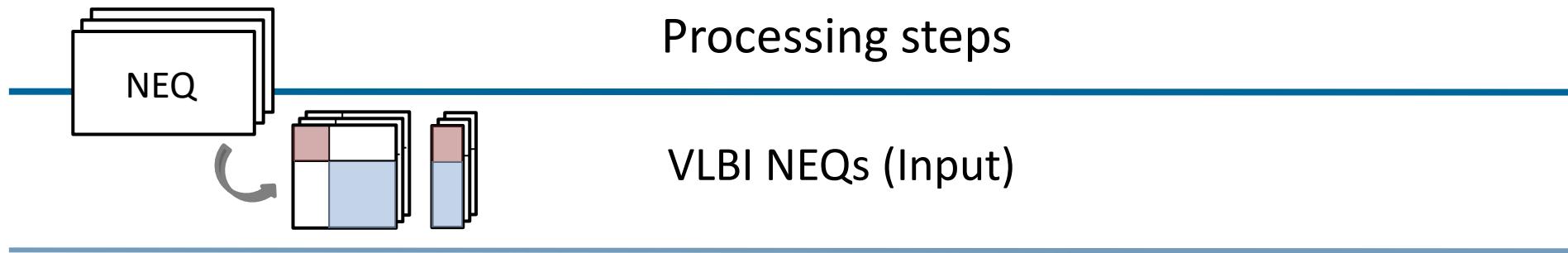
# Motivation

- Very Long Baseline Interferometry
  - Crucial for determination of CRF, scale, EOP (UT1, nutation)
- Vienna IVS Analysis Center
  - Global solution (multi-session solution) on NEQ level  
→ VLBI-only TRF + CRF solutions



# Global solution theory

e.g., Brockmann (1997), Angermann et al. (2004)



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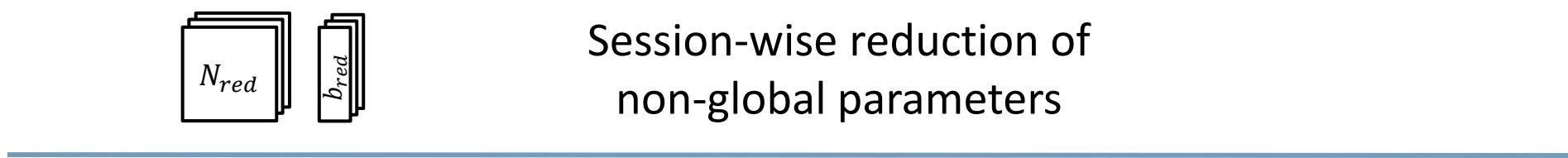
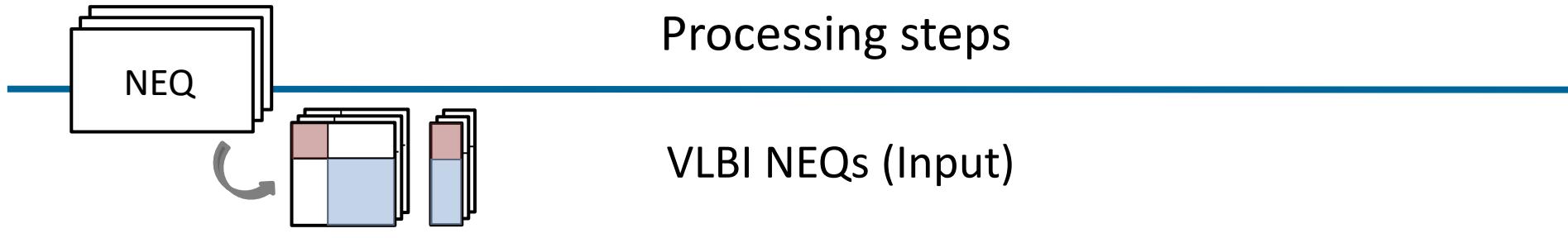
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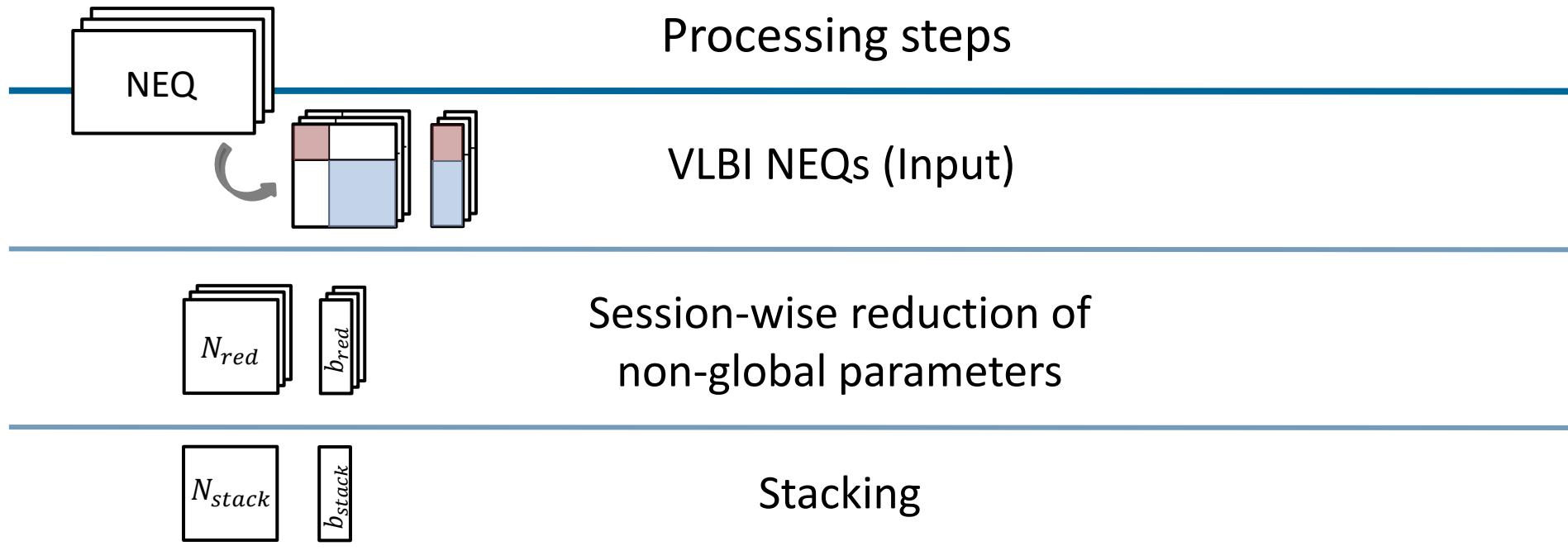
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e.g., Brockmann (1997), Angermann et al. (2004)



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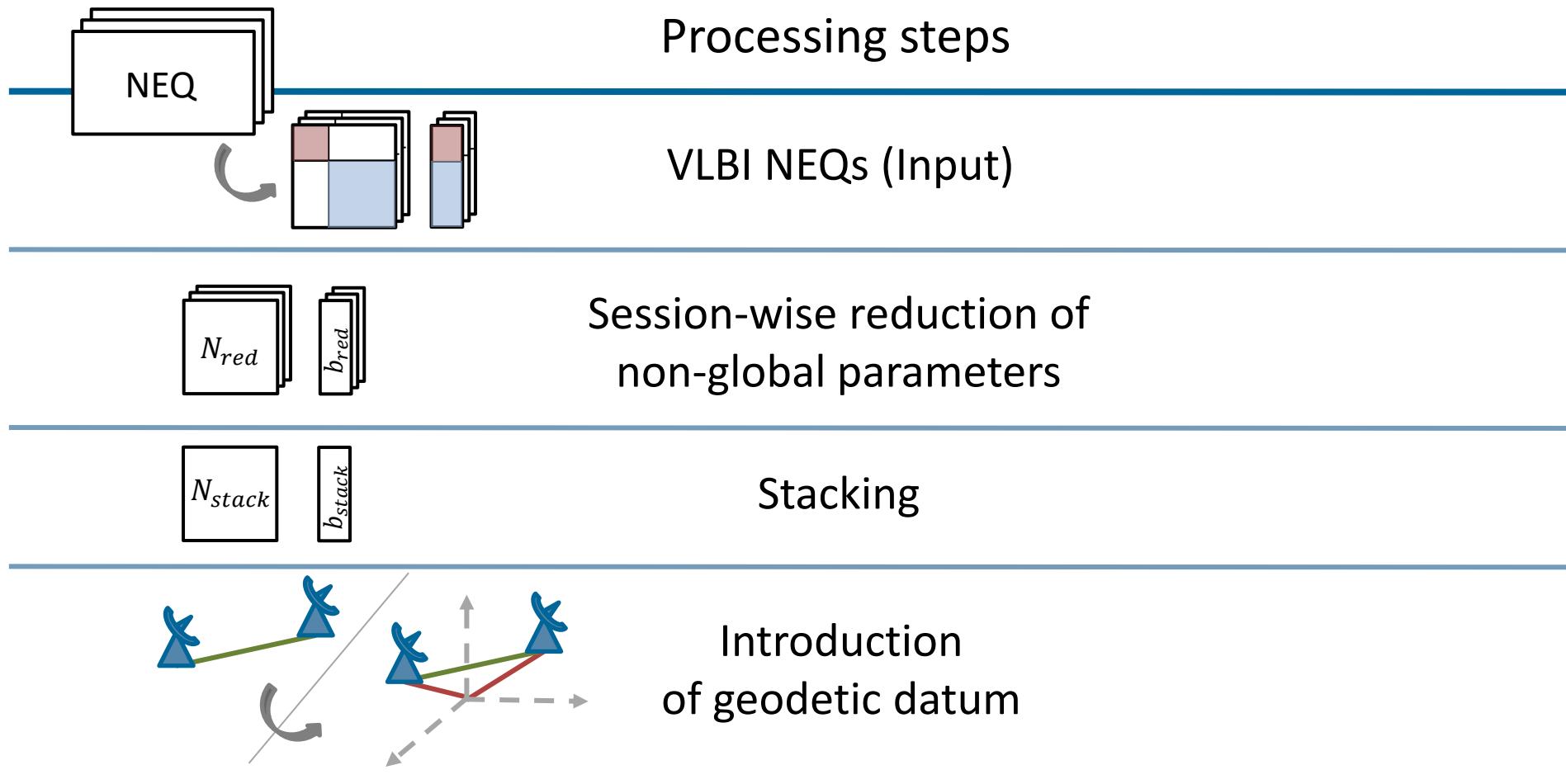
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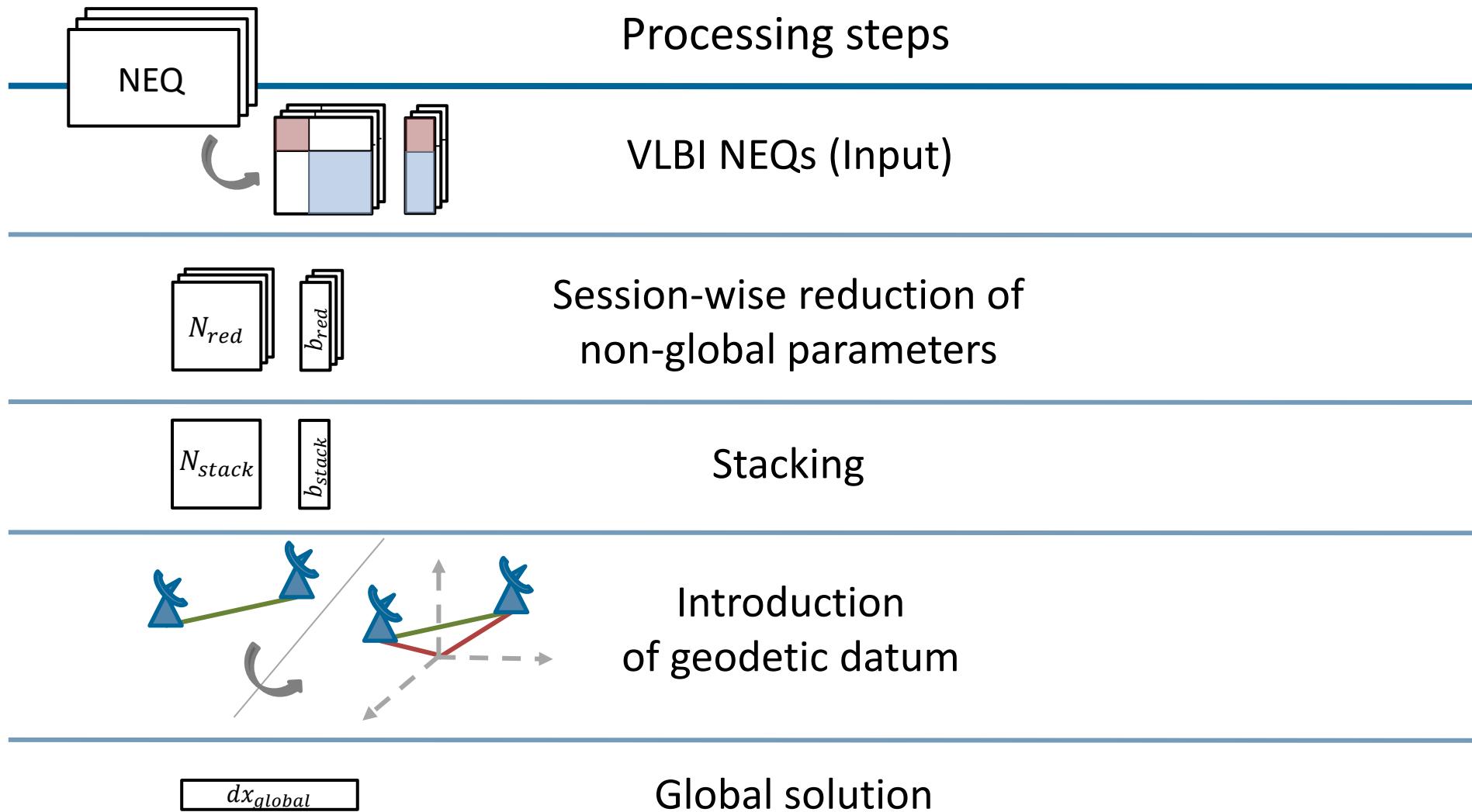
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# Global solution theory

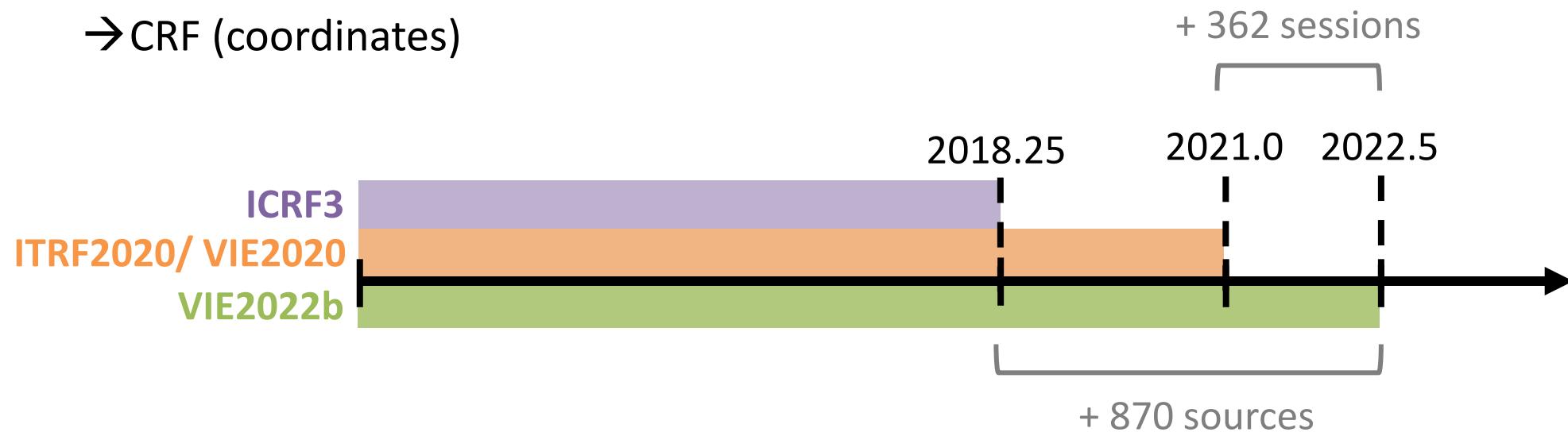
e.g., Brockmann (1997), Angermann et al. (2004)



# Global solution at VIE AC – VIE2020

Krásná et al., 2023 – revision submitted

- VIE IVS Analysis Center participated in efforts of calculating **ITRF2020**
- VIE VLBI-only solutions: **VIE2020** & **VIE2022b**
  - TRF (coordinates + linear velocities)
  - CRF (coordinates)



# Global solution at VIE AC – VIE2020

Krásná et al., 2023 – revision submitted

## Terrestrial reference frame

### – Transformation parameter from ITRF2020 to VIE2020/ VIE2022b

Motivation	$T_x$ [mm]	$T_y$ [mm]	$T_z$ [mm]	$R_x$ [ $\mu\text{as}$ ]	$R_y$ [ $\mu\text{as}$ ]	$R_z$ [ $\mu\text{as}$ ]	scale s [ppb] ([mm])
Global solution theory	$\dot{T}_x$ [mm/yr]	$\dot{T}_y$ [mm/yr]	$\dot{T}_z$ [mm/yr]	$\dot{R}_x$ [ $\mu\text{as}/\text{yr}$ ]	$\dot{R}_y$ [ $\mu\text{as}/\text{yr}$ ]	$\dot{R}_z$ [ $\mu\text{as}/\text{yr}$ ]	$\dot{s}$ [ppb/yr] ([mm/yr])
VIE2020	$-0.8 \pm 1.7$	$-3.4 \pm 1.7$	$-2.0 \pm 1.7$	$60.5 \pm 69.2$	$21.6 \pm 68.5$	$-5.4 \pm 54.2$	$0.56 \pm 0.26 (3.6 \pm 1.7)$
VIE2022b	$0.0 \pm 0.0$	$-0.2 \pm 0.0$	$0.0 \pm 0.0$	$6.0 \pm 1.7$	$0.8 \pm 1.8$	$4.4 \pm 1.5$	$0.02 \pm 0.01 (0.1 \pm 0.0)$

→ Good agreement between ITRF and VLBI-only VIE TRF solution

# Global solution at VIE AC – VIE2020

Krásná et al., 2023 – revision submitted

## Terrestrial reference frame

- Transformation parameter from ITRF2020 to VIE2020/ VIE2022b

	$T_x$ [mm]	$T_y$ [mm]	$T_z$ [mm]	$R_x$ [ $\mu\text{as}$ ]	$R_y$ [ $\mu\text{as}$ ]	$R_z$ [ $\mu\text{as}$ ]	scale s [ppb] ([mm])
	$\dot{T}_x$ [mm/yr]	$\dot{T}_y$ [mm/yr]	$\dot{T}_z$ [mm/yr]	$\dot{R}_x$ [ $\mu\text{as}/\text{yr}$ ]	$\dot{R}_y$ [ $\mu\text{as}/\text{yr}$ ]	$\dot{R}_z$ [ $\mu\text{as}/\text{yr}$ ]	$\dot{s}$ [ppb/yr] ([mm/yr])
VIE2020	$-0.8 \pm 1.7$	$-3.4 \pm 1.7$	$-2.0 \pm 1.7$	$60.5 \pm 69.2$	$21.6 \pm 68.5$	$-5.4 \pm 54.2$	$0.56 \pm 0.26$ ( $3.6 \pm 1.7$ )
	$0.0 \pm 0.0$	$-0.2 \pm 0.0$	$0.0 \pm 0.0$	$6.0 \pm 1.7$	$0.8 \pm 1.8$	$4.4 \pm 1.5$	$0.02 \pm 0.01$ ( $0.1 \pm 0.0$ )
VIE2022b	$1.9 \pm 0.7$	$-1.0 \pm 0.7$	$-2.3 \pm 0.7$	$35.4 \pm 29.4$	$36.2 \pm 29.0$	$21.4 \pm 23.2$	$0.59 \pm 0.11$ ( $3.7 \pm 0.7$ )
	$0.2 \pm 0.0$	$-0.2 \pm 0.0$	$-0.2 \pm 0.0$	$5.6 \pm 0.7$	$4.3 \pm 0.7$	$1.8 \pm 0.6$	$0.03 \pm 0.00$ ( $0.2 \pm 0.0$ )

→ Good agreement between ITRF and VLBI-only VIE TRF solution

- Scale difference
  - Trend after 2014
- VLBI working group established

# Global solution at VIE AC – VIE2020

Krásná et al., 2023 – revision submitted

## Terrestrial reference frame

### – Antenna height

**VIE2022b** vs. ITRF2020:

→ estimated heights are higher  
in VIE2022b (scale difference)

→ lower height error

**VIE2022b** vs. **VIE2020**:

→ lower height error

antenna	$\Delta h \pm m_{\Delta h}$ [mm]	$m_h$ [mm]	
		VIE2022b (VIE2020)	ITRF2020
FD-VLBA	1.6 ± 1.4	0.2 (0.2)	1.4
GGAO12M	7.7 ± 3.5	0.7 (1.3)	3.4
GGAO7108	-77.3 ± 30.6	10.6 (10.7)	28.7
HRAS_085	33.0 ± 17.4	17.1 (17.1)	3.3
KOKEE	-0.8 ± 1.2	0.2 (0.2)	1.2
KOKEE12M	-1.6 ± 3.6	0.4 (0.6)	3.5
MACGO12M	16.3 ± 11.2	0.4 (x.x)	11.2
NYALE13S	-2.2 ± 27.3	0.8 (2.1)	27.3
NYALES20	4.7 ± 1.1	0.2 (0.2)	1.1
ONSA13NE	2.6 ± 3.1	0.2 (0.4)	3.1
ONSA13SW	4.2 ± 4.0	0.3 (0.5)	4.0
ONSALA60	1.4 ± 1.0	0.2 (0.2)	1.0
RAEGSMAR	-21.7 ± 390.9	8.6 (x.x)	390.8
RAEGYEB	9.9 ± 2.0	0.4 (0.5)	1.9
WETTZ13N	3.2 ± 1.1	0.2 (0.2)	1.1
WETTZ13S	4.0 ± 1.5	0.3 (0.4)	1.5
WETTZELL	4.2 ± 0.9	0.1 (0.1)	0.9
YEBES40M	2.2 ± 1.3	0.2 (0.2)	1.3

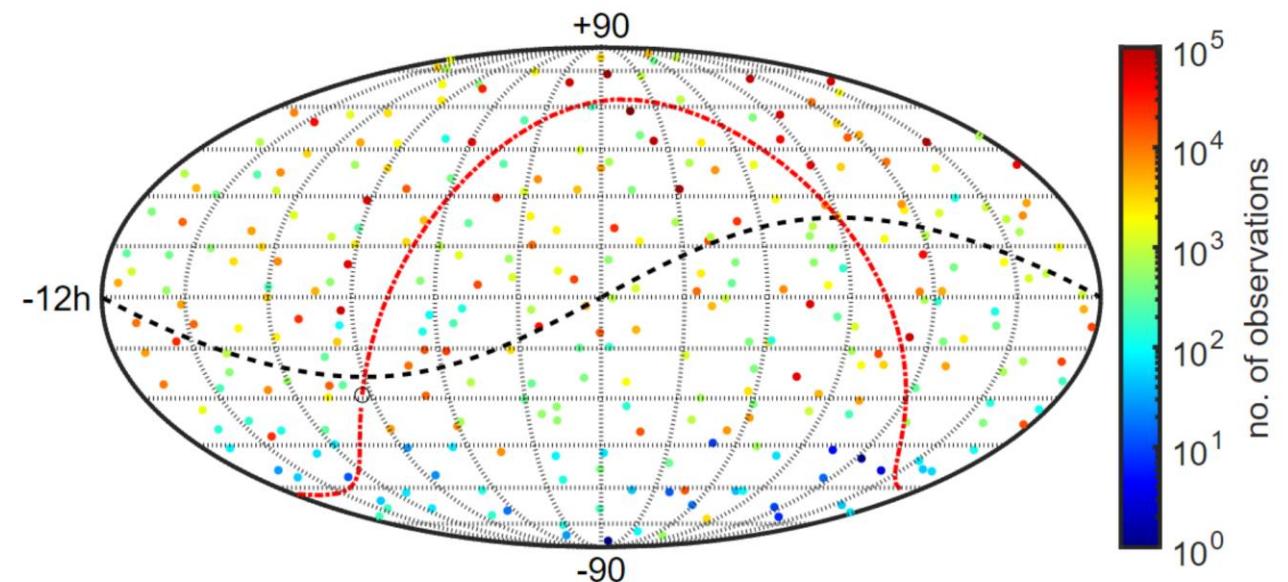
Antenna height differences and formal errors

# Global solution at VIE AC – VIE2020

Krásná et al., 2023 – revision submitted

## Celestial reference frame

- **VIE2022b-sx** (5407 sources = ICRF3 + 870 sources)
- Unweighted NNR on 301 ICRF3 defining sources



Number of observations of **ICRF3 defining sources** from ICRF3  
cutoff date (MAR 2018) – VIE2022b cutoff date (JUN 2022)

# Global solution at VIE AC – VIE2020

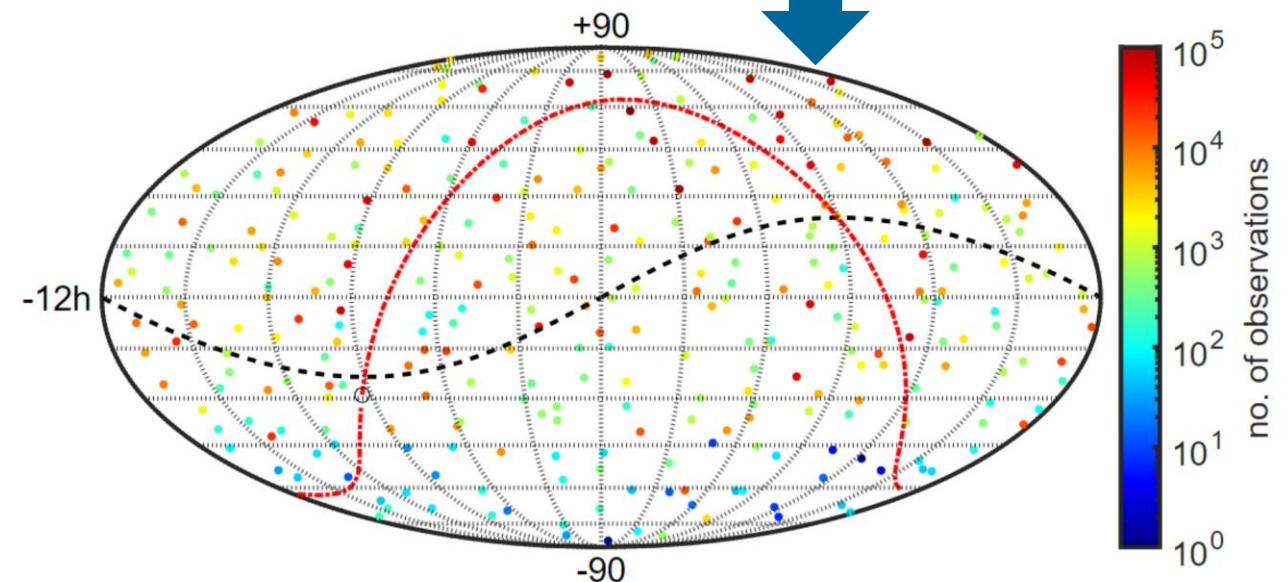
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\*IVS-CRDS (de Witt et al. 2019);  
AOV (McCallum et al. 2019);  
AUS mixed-mode program (McCallum et al. 2022)

## Celestial reference frame

- **VIE2022b-sx** (5407 sources = ICRF3 + 870 sources)
- Unweighted NNR on 301 ICRF3 defining sources

Despite great international effort\*  
→ increase in *#observations* slower  
for southern defining sources



Number of observations of **ICRF3 defining sources** from ICRF3  
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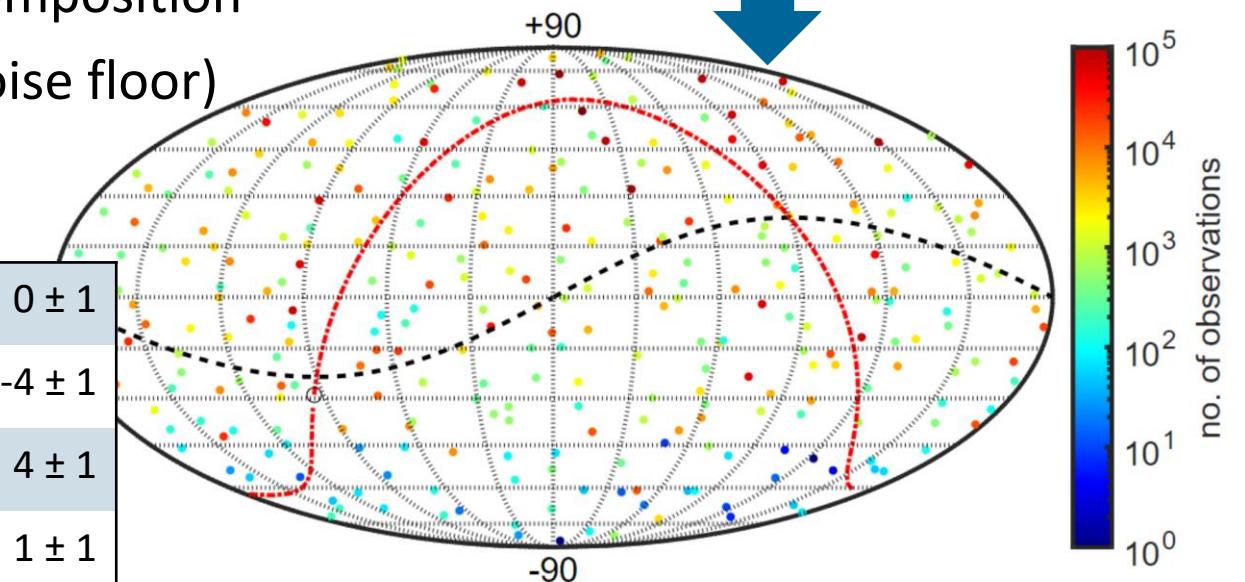
## Celestial reference frame

- **VIE2022b-sx** (5407 sources = ICRF3 + 870 sources)
- Unweighted NNR on 301 ICRF3 defining sources
- Vector spherical harmonics decomposition
- Differences < 30  $\mu\text{as}$  (below noise floor)

Despite great international effort\*  
→ increase in #observations slower  
for southern defining sources

VSH parameters [ $\mu\text{as}$ ]

$R_1$	$9 \pm 2$	$a_{2,0}^e$	$20 \pm 3$	$a_{2,2}^{e,Re}$	$0 \pm 1$
$R_2$	$-9 \pm 2$	$a_{2,0}^m$	$6 \pm 2$	$a_{2,2}^{e,Im}$	$-4 \pm 1$
$R_3$	$-13 \pm 2$	$a_{2,1}^{e,Re}$	$-3 \pm 3$	$a_{2,2}^{m,Re}$	$4 \pm 1$
$D_1$	$1 \pm 2$	$a_{2,1}^{e,Im}$	$1 \pm 3$	$a_{2,2}^{m,Im}$	$1 \pm 1$
$D_2$	$-6 \pm 2$	$a_{2,1}^{m,Re}$	$-2 \pm 3$		
$D_3$	$-25 \pm 2$	$a_{2,1}^{m,Im}$	$3 \pm 3$		



# Global solution – TRF datum definition

- Motivation:
  - Development of a new **state-of-the-art/ stand-alone** Python software for intra-/ inter- technique combination on NEQ level
  - Input: pre-reduced, datum-free NEQ (SINEX)
- Status:
  - VLBI Global solution on NEQ level

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Global solution theory

Global solution at VIE AC

Global solution datum definition

Conclusion & Outlook

# Global solution – TRF datum definition

- Methods of introducing NNT/ NNR conditions
  - *Conditions with infinite weight (Helmert rendering)*  
= forcing translations/ rotations w.r.t. reference frame to be zero

e.g., Sillard and Boucher (2001)

$$N_c = \begin{bmatrix} N & B \\ B^T & 0 \end{bmatrix}$$

- *Constraints with covariances*  
= loose constraints: lead to a “loose fit” of network within the datum definition

e.g., Altamimi et al. (2002)

$$N_c = N + H^T PH \text{ with } H = (B^T B)^{-1} B^T$$

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# Global solution – TRF datum definition

- Methods of introducing NNT/ NNR conditions
  - *Conditions with infinite weight (Helmert rendering)*
  - *Constraints with covariances*

$$N_c = \begin{bmatrix} N & B \\ B^T & 0 \end{bmatrix}$$

$$N_c = N + H^T PH \text{ with } H = (B^T B)^{-1} B^T$$

- Scaling of  $B$

- strict:  $B_{NNT}$  by  $\sqrt{N}$  and  $B_{NNR}$  by  $\sqrt{\sum_{i=1}^N x_{0,i}^2 + y_{0,i}^2 + z_{0,i}^2}$
- common in VLBI community:  $B_{NNR}$  by  $R_E$

→ No impact on estimates/ variances, but on fulfillment of conditions

# Global solution – TRF datum definition

- Methods of introducing NNT/ NNR conditions
  - *Conditions with infinite weight (Helmert rendering)*
  - *Constraints with covariances*

$$N_c = \begin{bmatrix} N & B \\ B^T & 0 \end{bmatrix}$$

$$N_c = N + H^T PH \text{ with } H = (B^T B)^{-1} B^T$$

- Weighting of constraints
  - Comparability of methods? What are „infinite weights“?

$$CONSwCOV_{w=10^8} \cong HR$$



$\equiv$  formal error = 0.1 mm

# Conclusion & Outlook

- Conclusion:
  - **VIE2020/ VIE2022b – ITRF2020:** good agreement,  
2022b crucial for new VGOS telescopes
  - **VIE2022b-sx – ICRF3:** differences in VSH parameters below noise floor
- Outlook:
  - Development of new state-of-the-art/ stand-alone Python software for intra-/ and inter-technique combination on NEQ level

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# References

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