



Dilution of Precision (DOP) Factors for Evaluating Observations to Galileo Satellites with VLBI

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Abstract

Installing a VLBI transmitter on Galileo satellites will allow observing satellites in parallel to quasars with Very Long Baseline Interferometry (VLBI) telescopes. This offers a variety of new applications such as the direct determination of the absolute orientation of the satellite constellation with respect to the International Celestial Reference Frame (ICRF) and the improvement of the Terrestrial Reference Frame (TRF) exploiting the possibilities of direct high precision tying of the different space geodetic equipment. In preparation of these observations by enhancing the capabilities of the VLBI scheduling program VieSched++, we perform an evaluation study of observations of a Galileo satellite employing Dilution of Precision (DOP) factors. The idea is to introduce DOP factors in the decision process of VieSched++ after a thorough assessment of DOP factors for individual parameters. In our study, we choose an existing network of VLBI Global Observing System (VGOS) type telescopes for observing Galileo satellite GSAT0212 within a 24 h arbitrary session. Preparing the DOP factor analysis, we first carry out a theoretical study to investigate the VLBI sensitivity to satellite orbit displacements in the local orbital frame with normal (radial), tangential and cross-track direction. This analysis shows that the highest sensitivity of a satellite observation is that of the tangential component if the direction of the satellite track is parallel to the direction of the observing baseline. A satellite observation is most sensitive towards the cross-track component if these two directions are orthogonal to each other. The DOP factor analysis itself is performed separating the satellite position again into its three components and adding a separate DOP factor for the UT1-UTC (dUT1) parameter. The periods, where satellite observations are possible, were determined using VieSched++. At a later stage, these DOP factors will be used as an optimization criterion for the scheduling process. The DOP factors of potential observations from the chosen VGOS network to GSAT0212 reach minimum DOP values of 27.13 in normal, 1.49 in tangential, and 1.67 in cross-track direction and 0.45 for determining dUT1. With these results, which have confirmed intuitive considerations on the relative magnitudes, we have laid the groundwork for using DOP factors as driving criteria in the scheduling process of Galileo satellites embedded in regular VLBI observations of quasars.

Keywords

Dilution of Precision (DOP) factors · Galileo · VGOS · VieSched++ · VLBI

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1 Introduction

Very Long Baseline Interferometry (VLBI) is a space geodetic technique which is contributing in the determination of the International Terrestrial Reference Frame (ITRF) (Altamimi et al. 2016) and is determining the International Celestial Reference Frame (ICRF; Charlot et al. 2020) and is uniquely able to provide the full set of Earth Orientation Parameters (EOP; Petit and Luzum 2010) by observing very distant radio sources. The mounting of a VLBI transmitter on board of Galileo satellites would allow to observe both, quasars and satellites, with VLBI telescopes in parallel (Fig. 1). This will bring new opportunities for improvements in the above products along with some challenges in the overall realization. To observe geodetic satellites with radio telescopes employed for routine geodetic VLBI observations, it is necessary that the satellites transmit signals that mimic the emission of quasars and other compact extra-galactic radio sources. This raises questions about technical aspects of generating and emitting an artificial signal on Galileo satellites for VLBI observations and the successful observation and correlation, as already discussed in McCallum et al. (2016) and Jaradat et al. (2021).

Observations to satellites with VLBI antennas bring a variety of new opportunities as they enable the connection of the satellite positions with the celestial reference frame. This allows the determination of the absolute orientation of the satellite constellation with respect to the ICRF and the comparison of the satellite's position with the position obtained by using other space geodetic techniques. Connecting the geodetic techniques on satellites via space-ties enables an improved determination of the Terrestrial Reference Frame (ITRF), provided the tie vectors on the satellites are measured and known with utmost accuracy.

So far only few experiments have been conducted tracking a satellite with VLBI by either directly observing Global Navigation Satellite System (GNSS) signals (Plank et al. 2017; Tornatore et al. 2014) or signals of a dedicated VLBI transmitter (Hellerschmied et al. 2018; Hellerschmied 2018). However, several simulation studies were carried out with regard to satellite tracking for the realization of inter-technique frame ties (Plank 2013; Plank et al. 2016) and for incorporating and improving co-location in space (Anderson et al. 2018; Herrera Pinzn and Rothacher 2020). The already existing laser reflector arrays on Galileo satellites allow the observation of the satellites with Satellite Laser Ranging (SLR). Therefore, Galileo satellites integrate both GNSS and SLR techniques, which formed the basis for different studies on the combination of

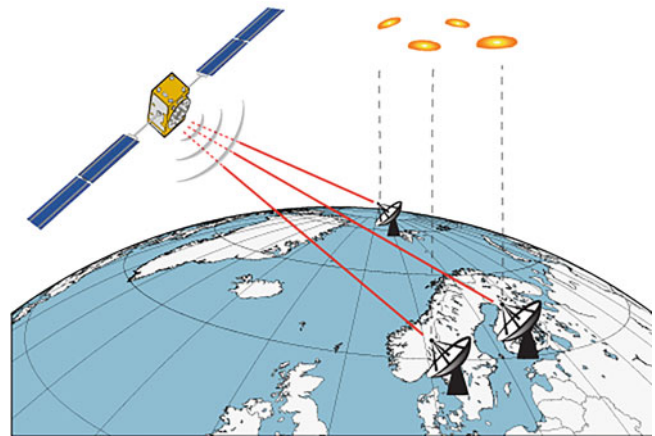


Fig. 1 Illustration of observations to satellites and quasars with VLBI telescopes

these observations (Thaller et al. 2011), using the satellite co-locations for determining precise Galileo orbits (Bury et al. 2021a) and realizing the geodetic datum (Bury et al. 2021b).

Further, combining VLBI observations of quasars and satellites in parallel allows Precise Orbit Determination (POD) of these Earth satellites for which the feasibility and potential of geodetic VLBI was examined before (Kłopotek et al. 2020; Mammadaliyev et al. 2021). In the course of a new project at TU Wien called VLBI2Galileo funded by the Austrian Science Fund (FWF), VLBI observations to Galileo satellites will be investigated in detail. The objectives of the project are the optimisation of the satellite scheduling process using Dilution of Precision (DOP) factors (Swanson 1978) as well as the implementation of orbit estimation from VLBI observations in the Vienna VLBI and Satellite Software (VieVS; Böhm et al. 2018).

2 Satellite Scheduling in VieSched++

Recently, the scheduling software VieSched++ (Schartner and Böhm 2019) has been extended by a satellite scheduling module (Wolf 2021) which allows the inclusion of satellite observations in a schedule among observations to quasars in a manual or automatic fashion. Depending on the satellite orbit and the chosen VLBI network, different observation periods are available during which a satellite is visible from more than two stations. Theoretically, potential observing times during these periods could be evaluated using DOP factors indicating the sensitivity of the VLBI observation towards specific parameters. In future this could be realized in VieSched++ by providing the values of the DOP factors to the scheduler as indicators and support in deciding which scan to schedule in case the satellite scans are scheduled

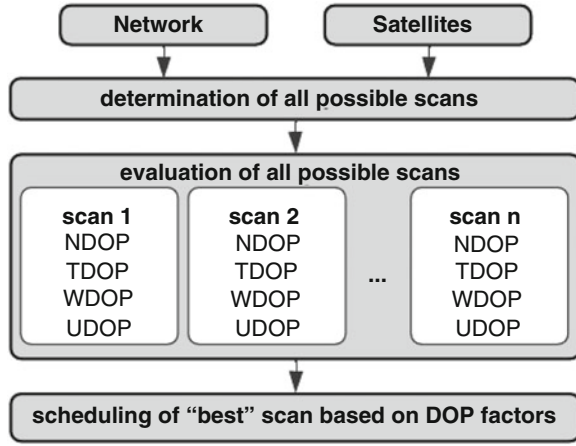


Fig. 2 Flowchart of using different DOP factors, i.e. normal DOP (NDOP), tangential DOP (TDOP), cross-track DOP (WDOP) and UT1-UTC DOP (UDOP), as weight factors during the scheduling of satellite scans

manually. If the satellite scans are scheduled automatically, the DOP factors represent an opportunity to be used as an optimization criterion during the scheduling process of a satellite scan. A possible implementation would be to determine all possible scans to the selected satellites from a chosen network and further evaluate those by the different DOP factors in order to finally select and schedule the best possible scan based on these indicators (Fig. 2).

3 Dilution of Precision Factors

Dilution of Precision factors indicate the change of the observable by a change of a specific parameter and therefore represent the sensitivity of the VLBI observation towards this parameter. The UT1-UTC Dilution of Precision (UDOP) factor is introduced by Belli (2020) and indicates the sensitivity of VLBI observations to satellites to UT1-UTC (dUT1). For the purpose of evaluating a satellite observation in the scheduling process with regard to its sensitivity to the position of the satellite, we introduce three further DOP factors. In the following, these DOP factors depicting the sensitivity of an observation towards the components of the satellite position in the local orbital frame with normal (N), tangential (T), and cross-track (W) direction (NTW-frame) are described. This orbital frame is aligned with the unit vectors

$$\mathbf{e}_N = \mathbf{e}_T \times \mathbf{e}_W, \quad \mathbf{e}_T = \frac{\dot{\mathbf{r}}}{\|\dot{\mathbf{r}}\|}, \quad \mathbf{e}_W = \frac{\mathbf{r} \times \dot{\mathbf{r}}}{\|\mathbf{r} \times \dot{\mathbf{r}}\|}, \quad (1)$$

which are defined by the Earth Centered Inertial (ECI) position \mathbf{r} and velocity $\dot{\mathbf{r}}$ vectors of the satellite. The unit vector \mathbf{e}_T is along the orbital velocity vector (tangential), \mathbf{e}_W

along the orbital angular momentum vector (cross-track) and \mathbf{e}_N is completing the right handed system.

The DOP factors correspond to the partial derivatives of a simplified model of an observation equation of the time delay in the Geocentric Celestial Reference System (GCRS; see Hellerschmied (2018) and the formalism described by Klioner (1991)) with respect to the satellite position in the GCRS which are formulated as

$$\begin{pmatrix} \frac{\partial \Delta\tau}{\partial x} \\ \frac{\partial \Delta\tau}{\partial y} \\ \frac{\partial \Delta\tau}{\partial z} \end{pmatrix} = \frac{1}{c} \left[\frac{\mathbf{L}_2}{\|\mathbf{L}_2\|} - \frac{\mathbf{L}_1}{\|\mathbf{L}_1\|} \right] - \frac{1}{c^2} \left[\dot{\boldsymbol{\omega}}_2 - \frac{\mathbf{L}_1}{\|\mathbf{L}_1\|\|\mathbf{L}_2\|} (\mathbf{L}_2 \cdot \dot{\boldsymbol{\omega}}_2) \right. \\ \left. + \|\mathbf{L}_1\| \frac{\mathbf{L}_2}{\|\mathbf{L}_2\|^3} (\mathbf{L}_2 \cdot \dot{\boldsymbol{\omega}}_2) - \frac{\|\mathbf{L}_1\|}{\|\mathbf{L}_2\|} \dot{\boldsymbol{\omega}}_2 \right], \quad (2)$$

where c denotes the speed of light, \mathbf{L}_1 and \mathbf{L}_2 the GCRS vectors between the satellite and the first and second VLBI station and $\dot{\boldsymbol{\omega}}_2$ represents the GCRS velocity vector of the second station. The scaling factor $(1 - L_G)$, where $L_G = 6.969290134 \times 10^{-10}$ is a defining constant (Petit and Luzum 2010), to transform the time delay from the Geocentric Coordinate Time (TCG) to the Terrestrial Time (TT) was neglected due to its rather small influence. Further, these partial derivatives are multiplied with the speed of light in order to transform their unit from time per length to dimensionless and a subsequent rotation into the orbit fixed satellite system (NTW-frame) is applied.

The partial derivatives of the time delay with respect to the satellite position in the orbital frame are determined for each baseline and represent the entries in the Jacobian matrix A . In this approach the matrix A is formed for each component separately, as it is shown for the normal component in Eq. 3.

The normal equation matrix N is calculated with $A^T P A$, where A^T denotes the transposed Jacobian matrix and P the weight matrix, which is in this work assumed as the unit matrix (Eq. 4). The cofactor matrix Q_{xx} is the inverse of N . Finally, the NDOP factor is calculated from the square root of Q_{xx} (Eq. 6) and therefore indicates the uncertainty in the normal component based on the geometrical configuration.

The approach outlined for the determination of the NDOP (Eqs. 3–5) is also applied to calculate the TDOP (Eq. 7) and WDOP (Eq. 8) using the partial derivatives of the time delay with respect to the tangential and cross-track component. The UDOP factor is determined in the same way as the other DOP factors but taking the sensitivity of the different VLBI satellite observations towards UT1-UTC into account. The mathematical derivation on how to determine the sensitivity towards UT1-UTC and further the UDOP can be found in Belli (2020).

These DOP factors represent the orbital error in the respective component per VLBI measurement error, both in

units of length. For example, the normal DOP refers to the normal orbital error.

$$A = \begin{pmatrix} \frac{\delta\Delta\tau_1}{\delta n} \\ \frac{\delta\Delta\tau_2}{\delta n} \\ \vdots \\ \frac{\delta\Delta\tau_n}{\delta n} \end{pmatrix} \quad (3)$$

$$N = A^T P A = \begin{pmatrix} \frac{\delta\Delta\tau_1}{\delta n} & \frac{\delta\Delta\tau_2}{\delta n} & \dots & \frac{\delta\Delta\tau_n}{\delta n} \end{pmatrix} \cdot \begin{pmatrix} \frac{\delta\Delta\tau_1}{\delta n} \\ \frac{\delta\Delta\tau_2}{\delta n} \\ \vdots \\ \frac{\delta\Delta\tau_n}{\delta n} \end{pmatrix} = \quad (4)$$

$$\left(\frac{\delta\Delta\tau_1}{\delta n}\right)^2 + \left(\frac{\delta\Delta\tau_2}{\delta n}\right)^2 + \dots + \left(\frac{\delta\Delta\tau_n}{\delta n}\right)^2 \quad (5)$$

$$Q_{xx} = N^{-1} = \frac{1}{\left(\frac{\delta\Delta\tau_1}{\delta n}\right)^2 + \left(\frac{\delta\Delta\tau_2}{\delta n}\right)^2 + \dots + \left(\frac{\delta\Delta\tau_n}{\delta n}\right)^2} \quad (6)$$

$$NDOP = \sqrt{\frac{1}{\left(\frac{\delta\Delta\tau_1}{\delta n}\right)^2 + \left(\frac{\delta\Delta\tau_2}{\delta n}\right)^2 + \dots + \left(\frac{\delta\Delta\tau_n}{\delta n}\right)^2}} \quad (7)$$

$$TDOP = \sqrt{\frac{1}{\left(\frac{\delta\Delta\tau_1}{\delta t}\right)^2 + \left(\frac{\delta\Delta\tau_2}{\delta t}\right)^2 + \dots + \left(\frac{\delta\Delta\tau_n}{\delta t}\right)^2}} \quad (7)$$

$$WDOP = \sqrt{\frac{1}{\left(\frac{\delta\Delta\tau_1}{\delta w}\right)^2 + \left(\frac{\delta\Delta\tau_2}{\delta w}\right)^2 + \dots + \left(\frac{\delta\Delta\tau_n}{\delta w}\right)^2}} \quad (8)$$

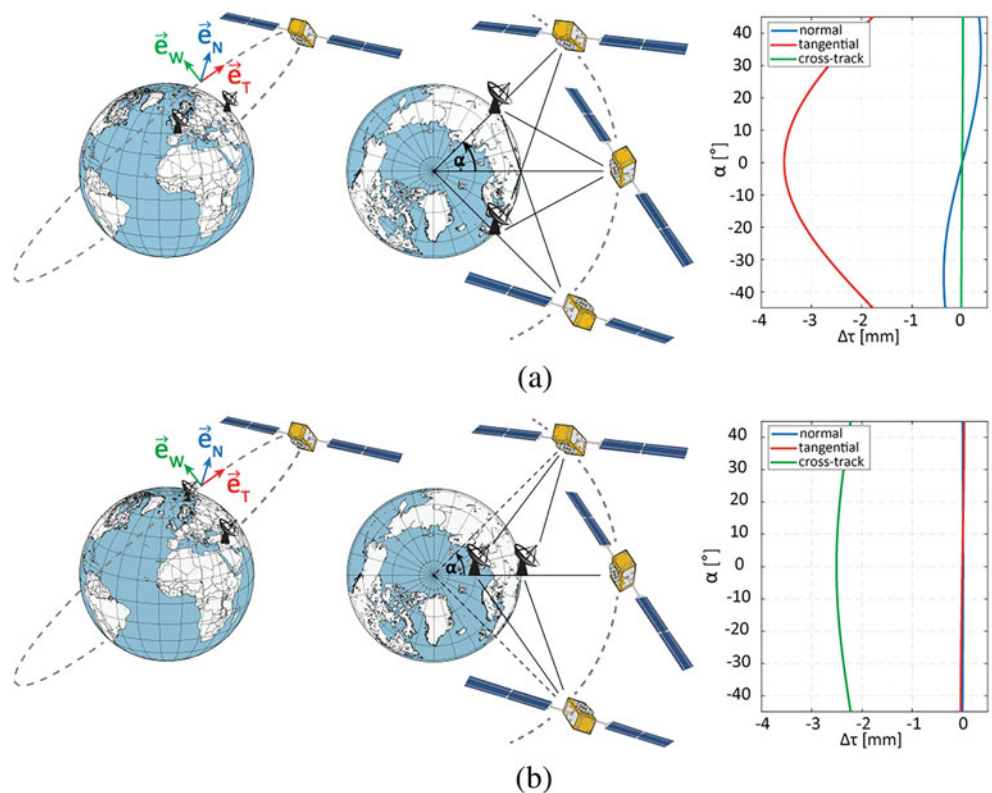
The smaller the value of the respective DOP, the higher the sensitivity of the observation towards the corresponding component and the better the geometric configuration of the satellite position with respect to the observing stations for determining this component.

4 Sensitivity of Satellite Observations

Different geometric constellations of satellites with respect to baselines produce different impacts of the VLBI observations on the satellite position parameters. To prepare for the DOP investigations (Sect. 5), we examine the sensitivity of satellite observations with VLBI to satellite orbit displacements. Here, the impact $\Delta\tau$ of a satellite orbit displacement by one centimeter in the normal, tangential and cross-track component on τ , the difference in arrival time of the radio signal between pairs of radio telescopes, is determined and depicted in Fig. 3.

Scenario (a) analyses the impact for observations of two stations at 45° latitude with a difference of 90° in longitude forming an east-west baseline. For this observation geometry, the VLBI observable is almost insensitive for the cross-track component (green) and hardly sensitive for the normal com-

Fig. 3 Illustration of two scenarios of two VLBI stations observing a satellite between -45° and 45° for the satellite position angle α defined in the illustration in the middle. The left plots illustrate the observing scenario. The right plots depict the impact $\Delta\tau$ of a 1 cm orbit displacement in normal (blue), tangential (red) and cross-track (green) direction on the observable τ . (a) Observations to a satellite from two VLBI stations forming an east-west baseline. (b) Observations to a satellite from two VLBI stations forming a north-south baseline



ponent (blue), especially if the satellite is exactly between the two stations, the impact of a change in normal direction becomes zero. The shift of the satellite in tangential direction (red) has a greater impact on the observable which can be as large as 3.5 mm.

Scenario (b) depicts two VLBI stations located at the same longitude being 40° apart in terms of latitude forming a north-south baseline. The impact on the observable for this constellation is insignificant if the satellite is shifted in the normal or tangential direction. However, this observation geometry is suitable to determine the cross-track component as the impact on the observable by a shift in this direction is greater than 2 mm.

This theoretical analysis shows that a satellite observation is most sensitive towards the tangential component if the direction of the satellite track is parallel to the observing baseline. The highest sensitivity towards the cross-track component arises if the direction of the satellite track is orthogonal to the observing baseline. The impact on the VLBI observable by a change in the normal component was significantly smaller for both observation setups, which therefore represents the component which is worse determinable as expected for a pseudorange measurement system.

5 Results

As a representative example, our investigations concentrate on one day of observations of the Galileo satellite GSAT0212 with a currently operational network of five VLBI Global Observing System (VGOS; Petrachenko et al. 2012) type stations (Fig. 4). For this network, we determined the DOP factors described in Sect. 3, i.e., normal DOP (NDOP), tangential DOP (TDOP), and cross-track DOP (WDOP), as well as the UT1-UTC DOP (UDOP). The analysis is carried out for 24 h starting on June 7, 2021 00:00:00 UTC (Fig. 5). Table 1 shows the minimum values of the determined DOP factors for this analysis.

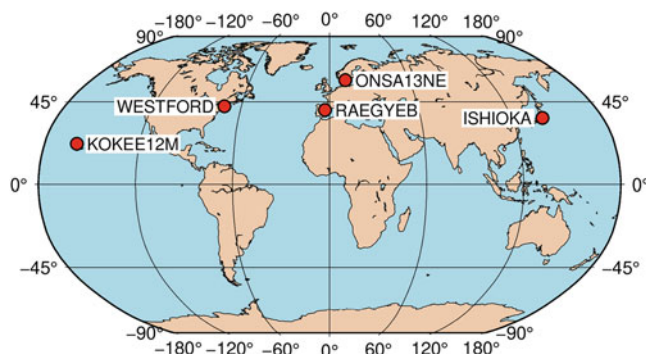


Fig. 4 VGOS stations considered in this study

Table 1 Minimum values of the DOP factors for the performed analysis

	NDOP	TDOP	WDOP	UDOP
GSAT0212 (E03)	27.13	1.49	1.67	0.45

5.1 Normal DOP Factor

The normal DOP factor indicates the sensitivity of a satellite observation to the normal (radial) component of the satellite position in the NTW-frame. It is evident that the values of the NDOP are in general higher compared to values of other DOP factors (Fig. 5b). By relating the values of the NDOP with the number of stations, for which the satellite is visible simultaneously, it can be stated that the values of the NDOP are lower if the satellite is visible from more stations at the same time. For example, the NDOP is below 60, when the satellite's trajectory is above North America and the satellite is visible for about 4.5 h from all five stations. When the satellite travels southwards and is only visible from two stations, the NDOP jumps to a value greater than 60. The same can be observed when the satellite is orbiting over Europe where it is visible for four stations simultaneously for 25 min (Fig. 5a). This position results in an NDOP value of 27.13, which is the minimum value for this study and therefore represents the best suitable satellite position for determining the normal component.

5.2 Tangential DOP Factor

The sensitivity of a satellite observation to the tangential component of the satellite position is represented by the TDOP factor. As it was described in Sect. 4, the values of the TDOP are lower if the direction of the satellite track is parallel to the observing baseline which represents an appropriate observation geometry for determining the tangential component (Fig. 5c). This can be recognized when the satellite is orbiting above Europe being visible for four stations simultaneously for 25 min (Fig. 5a). For this and the further positions the TDOP value drops from more than five to a value below four as the satellite becomes visible for the station ISHIOKA and the direction of the baseline between ONSA13NE and ISHIOKA is similar to the direction of the satellite track. For the performed analysis the best observation geometry for determining the tangential component reaches a value of 1.49 for the TDOP factor.

5.3 Cross-Track DOP Factor

The cross-track DOP factor indicates the sensitivity of a satellite observation with VLBI to the cross-track component

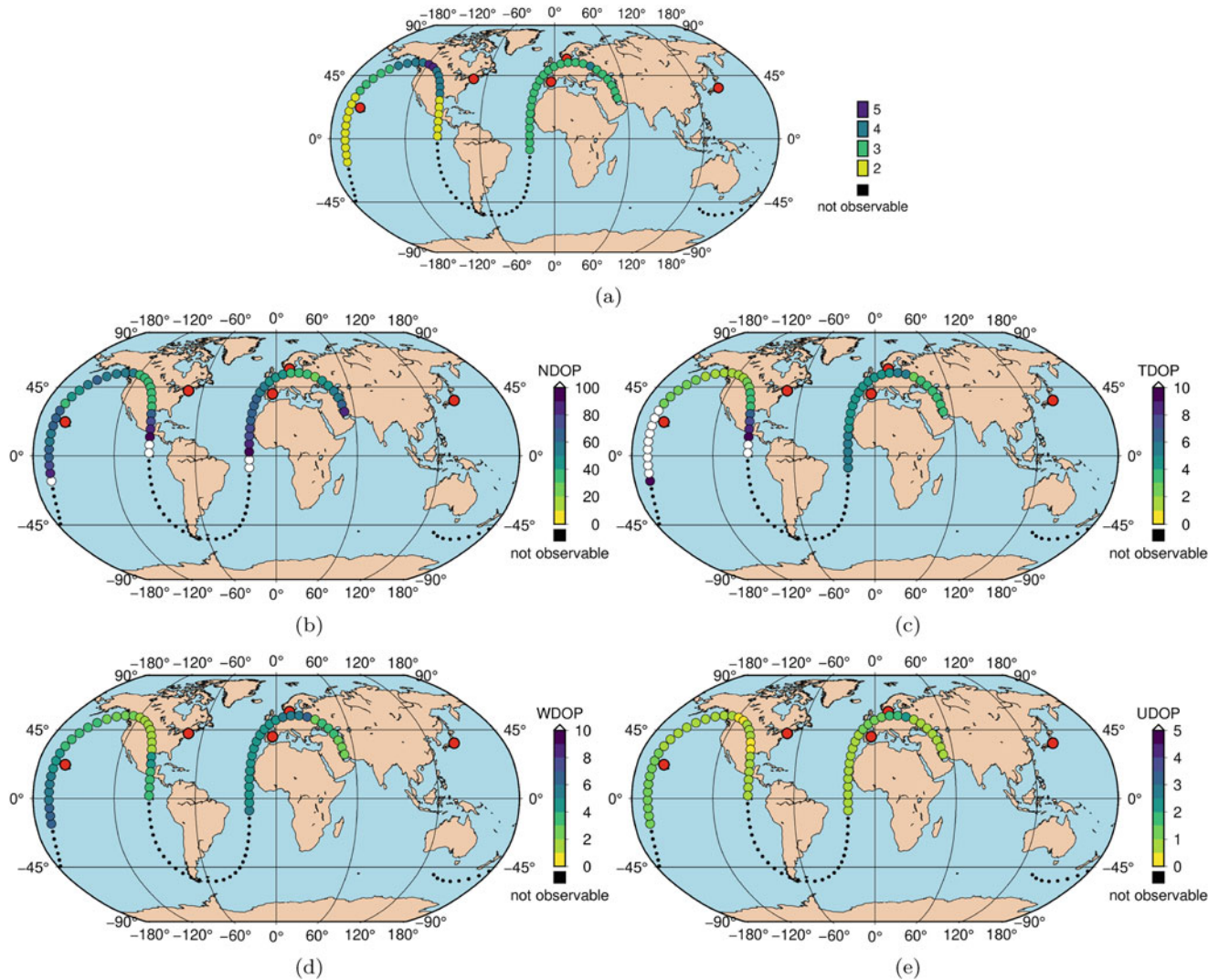


Fig. 5 Ground track of the satellite GSAT0212 (E03) during 24 h starting on June 7, 2021 00:00:00 UTC. The dots represent the position of the satellite over Earth in a 15 min interval color-coded by (a) number of stations from which the satellite is visible and by the values of the (b)

NDOP, (c) TDOP, (d) WDOP and (e) UDOP factor. The smaller black dots represent positions where the satellite is not visible from at least two stations and the white circles represent positions with DOP values exceeding the scale

of the satellite position in the local orbital frame. The results in Fig. 5d confirm the outcomes of the theoretical study described in Sect. 4 as the values of WDOP are lower and an observation is therefore more sensitive towards the cross-track component if the direction of the satellite track is orthogonal to the observing baseline and the satellite passes through the observing stations. This is clearly visible when the satellite travels over North America where the WDOP value is below two and the satellite is visible for four or five stations simultaneously. Within these positions a minimum value of 1.67 is reached. When the satellite is travelling further southwards and is not visible anymore for the two stations in Europe the WDOP reaches values above three as the direction of the remaining baseline between KOKEE12M and WESTFORD and the direction of the

satellite track for these positions are not close to orthogonal anymore.

5.4 UT1-UTC DOP Factor

The UDOP factor represents the sensitivity of satellite observations to UT1-UTC. More precisely, it constitutes the UT1-UTC error on the Earth equator per VLBI measurement error, both in units of length and is therefore dimensionless. The results depict that the UDOP factor reaches the smallest values compared to the other DOP factors (Fig. 5e). The best observation geometry for determining dUT1 occurs when the satellite passes through stations forming baselines with a long east-west extension which results in a minimum value of

0.45 for this study. A visibility from stations forming north-south baselines correspond to a less sensitive observation geometry for dUT1. The minimum value is reached when the satellite is visible for three stations simultaneously and it is passing between these stations, namely KOKEE12M, WESTFORD and RAEGYEB. The impact of the observation geometry between the satellite and the observing stations on the UDOP factor can also be recognized when the satellite passes over Europe. Before the satellite becomes visible for four stations for a short period, becoming observable for the station ISHIOKA, the value of the UDOP factor is greater than one. Even though the satellite is visible for an east-west baseline for these positions it is not passing between these two stations. As soon as the satellite becomes visible for the station ISHIOKA the UDOP value drops below one as the satellite is now orbiting through stations forming a baseline with a long east-west extension.

6 Conclusion

In this study, all potential observation periods on June 7, 2021 to the Galileo satellite GSAT0212 from a network including five VGOS type stations were determined and investigated in terms of the Dilution of Precision (DOP) factors regarding the satellite position in the NTW-frame, namely normal DOP, tangential DOP and cross-track DOP, as well as the UT1-UTC DOP. The performed analysis has confirmed the well-known fact that the normal component is significantly worse determinable compared to the tangential and cross-track component. In order to investigate different approaches for precise orbit determination in future, the normal component may have to be constrained in the analysis stage by either using the ranging information of SLR observations or constraining it to the a-priori orbit information. A suitable observation geometry for determining the tangential component arises if the satellite track direction is parallel to the baseline between the observing stations. This reflects that the highest sensitivity is obtained in the tangential direction in a plane containing the baseline. In contrast, the cross-track component can be well determined if the direction of the satellite track is orthogonal to the observing baseline. A highly sensitive observation geometry with respect to dUT1 is constituted if the satellite passes between two stations forming a baseline with a long east-west extension. With these investigations, we have demonstrated that the DOP factors introduced here are suitable quantities for the evaluation and preferential selection of satellite observing geometries. As a consequence, we will use them for optimising the scheduling of satellite observations in VieSched++. The findings of this study are also applicable to legacy S/X and combinations of VGOS and legacy S/X VLBI systems, as well as to other satellite systems. Finally, DOP factors are not

only suitable for individual observations but could possibly also be introduced as a factor for characterizing the quality of the whole observing schedule for the determination of the absolute orientation of a satellite constellation.

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