Reference epochs in VLBI estimations of clock parameters

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Abstract By default, the reference epoch of the relative clock offsets has always been the start of the sessions. If we consider a simple first order model for the relative clock parameters, i.e., just an offset and a rate, the formal errors of the parameters improve if the reference epoch is chosen to be at the middle of the session. In Altamimi et al. (2002), this is called the epoch of minimum variance. In a small study we have investigated whether this fact could be exploited in VLBI data analysis. For the CONT17-L2 series of sessions, we ran solutions with the reference epochs of the clocks being the middle of the session and compared the results to those of standard solutions. We found that even for more sophisticated parameterizations of the clocks, i.e., with polynomials and piece-wise linear polygons, the formal errors of all clock parameters did improve significantly. Although the correlation matrices change as well, there, unfortunately, is no improvement for the formal errors of any other, (non-clock) parameter. At the same time, the condition numbers of the solutions did not change significantly either. We conclude that the effects of changing the reference epoch in VLBI estimations of clock parameters are confined to the clock parameter space alone.

Keywords VLBI clock parameters, epoch of minimal variance

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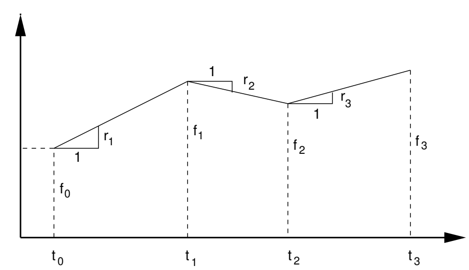
1 Introduction

In most analysis packages using least squares adjustments in Gauß-Markov models, the clock parameters are modelled with a second order polynomial and a superimposed series of piece-wise linear polygons (Fig. 1). The polygons can be formulated in two different ways. The first one is an initial offset at a reference epoch t_0 plus a new rate parameter for every segment of predefined duration (e.g., 20, 30 or 60 minutes). The functional values for each observation epoch t can be formulated according to Eq. 1 depending on the rates r_i and the segment limits t_i. This also serves as (part of) the VLBI observation equation which is the basis for the partial derivatives of the offset $f(t_0)$ and rate parameters r_i.



$$f(t) = f(t_0) + r_1(t_1 - t_0) + r_2(t_2 - t_1) + \ldots + r_n(t - t_{n-1}) \quad (1)$$

The rates also depend on the functional values f_i at the limits of the intervals.

$$r_i = \frac{f(t_i) - f(t_{i-1})}{t_i - t_{i-1}} \tag{2}$$

Consequently, we can introduce Eq. 2 in 1 and get a functional model purely based on the functional values (offsets) f_i (Eq. 3).

$$\begin{aligned} f(t) = f(t_0) & \\ &+ \frac{f(t_1) - f(t_0)}{t_1 - t_0}(t_1 - t_0) \\ &+ \frac{f(t_2) - f(t_1)}{t_2 - t_1}(t_2 - t_1) \\ &+ \frac{f(t_{i+1}) - f(t_i)}{t_{i+1} - t_i}(t - t_i) \\ &+ \ldots \end{aligned} \tag{3}$$

From this we can derive the partial derivatives for the offsets at the interval boundaries for the least squares adjustment. They have the form

$$\frac{\partial f}{\partial f_{i-1}} = \begin{cases} 1 - \frac{t - t_{i-1}}{t_i - t_{i-1}} & \text{for } t_{i-1} < t < t_i \\ 0 & \text{for all other epochs} \end{cases}$$

$$\frac{\partial f}{\partial f_i} = \begin{cases} \frac{t - t_{i-1}}{t_i - t_{i-1}} & \text{for } t_i < t < t_{i+1} \\ 0 & \text{for all other epochs} \end{cases}$$

This leads to a block diagonal scheme of the partials in the design matrix as depicted in Fig. 1.

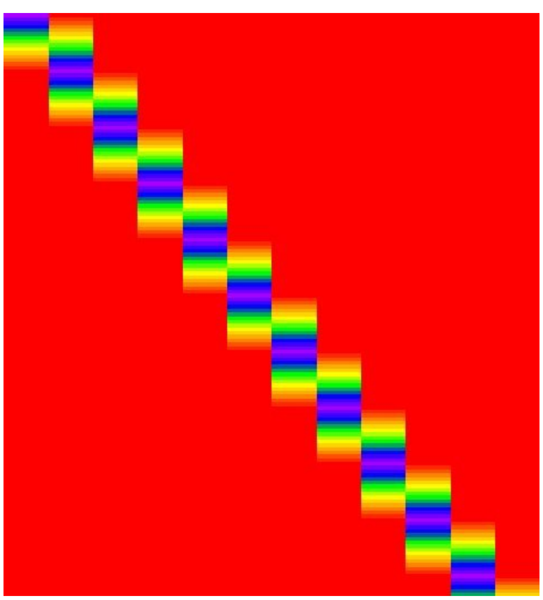


Fig. 1 Schematic distribution of magnitudes of partial derivatives for piece-wise linear offsets (red = 0, blue = 1)

The approach using the functional values or offsets (Eq. 3) is equivalent to that applying and estimating the rates (Eq. 1).

For convenience, the reference epoch of the first relative clock offset parameter $f(t_0)$ has always been set to the start of the respective session in almost all VLBI analysis packages. This has the consequence that, due to error propagation, the formal errors of the estimated clock offsets increase with time as can be seen in Fig. 2 (bottom/red) part. This obvious deficit of the piece-wise linear offset adjustment triggered the idea to investigate the effects of changes in the reference epoch.

We started with a simple fact of the estimation process. If we consider a first order model, i.e., just an offset and a rate, for observations ordered in time, such as the relative clock parameters, the formal errors of the regression parameters improve if the reference epoch is chosen to be at the middle of the session. In Altamimi et al. (2002), this is called the epoch of minimum variance. The question was whether the results of the VLBI parameter estimation change if we select the mean epoch instead of the beginning of the session for the clock offset parameters.

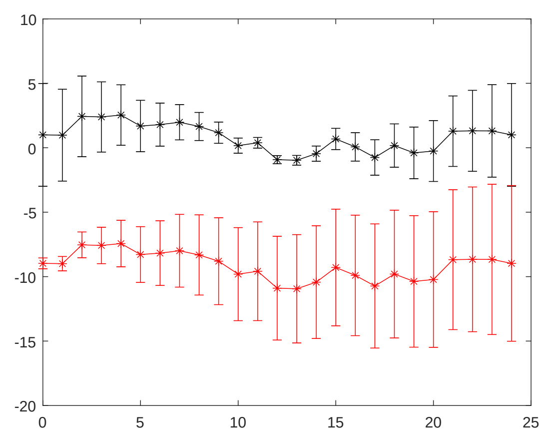


Fig. 2 Estimates [cm] of clock piece-wise linear polygons for OV-VLBA in session 17NOV28XA. 1 cm ≙ 33 ps. Units of x axis are hours since the session start. Bottom = Reference epoch at beginning of session, Top = reference epoch at middle of session.

2 Analysis

In a small study we have investigated whether the selection of a different clock reference epoch could be exploited in VLBI data analysis. For the CONT17-L2 series of sessions (Behrend et al., 2020), we ran solutions with the reference epochs of the clocks being the middle of the session and compared the results to those of standard solutions. The clock parameterization for all stations except of a reference station consists of a second order clock polynomial plus piece-wise linear segments of 1 hour duration represented as a clock offset polygon. Zenith wet delays and gradients, all EOP, station coordinate offsets constraint by NNR/NNT conditions and source position offsets constraint by NNR conditions are the other parameters.

3 Results

We found that for simple clock polynomials as well as for any standard parameterizations of the clocks, e.g., with polynomials and piece-wise linear polygons, the absolute values of the offsets changed but not the values of the other parameters. For the piece-wise linear polygons, the differences changed in the form of a constant bias for every epoch.

The most noticeable effect is that the formal errors of all clock offset parameters do improve significantly (see example Fig. 2). We see that the monotonous deterioration of the formal errors is restricted by the reference epoch lying in the middle of the session. Significant improvements also apply to the formal errors of all clock rates but not to those of the quadratic terms which remain unchanged.

For the changes in the correlation matrices, we show this for session 17NOV28XA as an example as well (Fig. 3). The first 297 parameters in the list are the clock parameters. We see station-wise block-diagonal patterns for the 273 clock offsets and distinct changes between the clock offsets and the 24 rate and quadratic terms of the clock functions. On the other hand there is a number of very small changes in the correlations of the clock offset parameters with other non-clock parameters (see also zoomed graph Fig. 4). Most noticeable are the changes for the zenith wet delays at the middle of the session. Within the block of non-clock parameters vs. non-clock parameters, no changes are discernible at all.

Although we had hoped for it, the changes in the correlations do not affect the formal errors of these other parameters beyond some small numerical differences. So, unfortunately, there is no improvement for the formal errors of any other, non-clock parameter.

We also looked at the condition numbers of the normal matrices but did not find any significant effects either.

4 Conclusions

Except of the fact that the formal errors of the clock offsets improve considerably, we did not find any evidence that the selection of the reference epoch for the clock offset estimates can improve VLBI solutions in general. We conclude that the effects of changing the reference epoch are confined to the clock offset and rate parameters alone.

References

Altamimi Z, Sillard P, Boucher C (2002) ITRF2000: A new release of the International Terrestrial Reference Frame for earth science applications. *JGR*, 107:B10, 2214, doi: 10.1029/2001JB000561.

Behrend D, Thomas C, Gipson J, Himwich E, Le Bail K (2020) On the organization of CONT17. *J Geod*, 94:100, doi: 10.1007/s00190-020-01436-x.

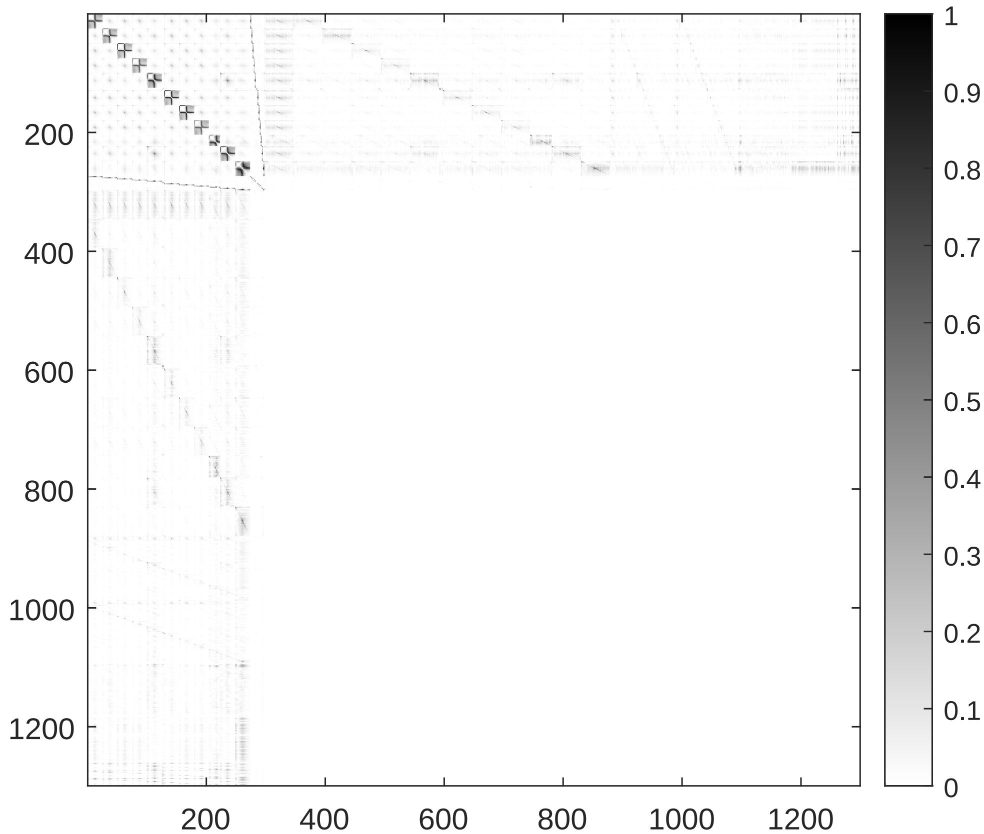


Fig. 3 Absolute differences in correlation coefficients in session 17NOV28XA. Order of parameters: 273 clock offsets, 24 rate and quad. terms of clock functions, 581 zenith wet delays, 216 gradients, 10 EOP and their rates, 156 source position components, and 39 station coordinate offsets.

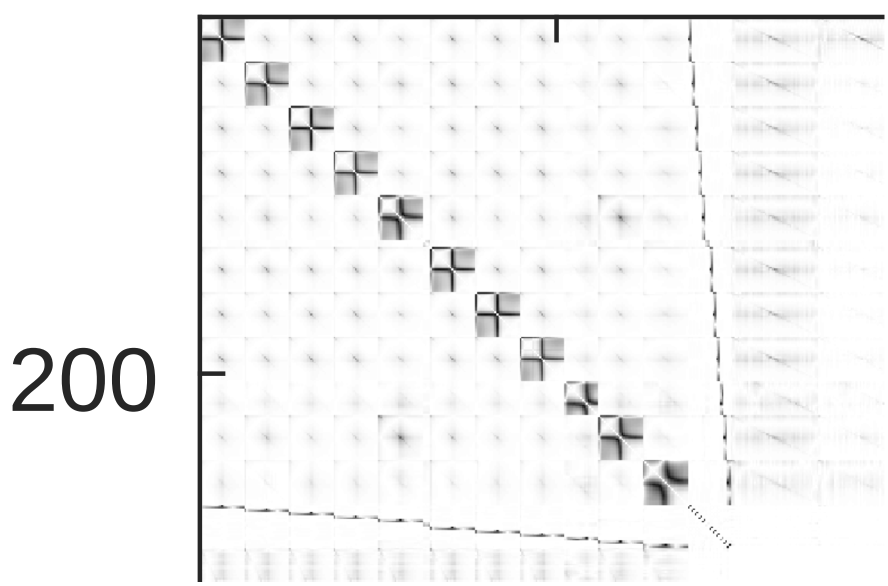


Fig. 4 Enlarged excerpt of Fig. 3 (Absolute differences in correlation coefficients).