Comparison of Integrated GNSS LOD to dUT1

M. Mikschi, J. Böhm, S. Böhm, D. Horozovic

Abstract The problem of estimating the bias of Length of Day (LOD) data from Global Navigation Satellite Systems (GNSS) is discussed. A method for estimating the bias is described and ideal parameters of the method for the used data are given. The time correction dUT1 is estimated using four different GNSS data sets and the achieved results are presented and discussed. For the best data set the root mean square error (RMSE) of the dUT1 estimate is 40 μs after 7 days and 60 μs after 14 days.

Keywords dUT1 · LOD · GNSS

1 Introduction

The time correction dUT1 is an important Earth orientation parameter that is regularly determined with Very-Long-Baseline Interferometry (VLBI) intensive sessions. While VLBI is the only technique that is capable of estimating dUT1 directly, its (negative) time derivative, Length of Day (LOD), can be measured with, among other techniques, a global GNSS network. Therefore, it is possible to estimate dUT1 by integrating GNSS LOD values and adding the known dUT1 value at t_0.

$$dUT1(T) = \int_{t_0}^{T} -LOD(t)dt + dUT1(t_0) \qquad (1)$$

However, GNSS LOD values contain a bias which has to be accounted for in order to achieve good dUT1 estimates. This bias mainly stems from perturbations of the GNSS orbits which cannot be differentiated from changes in the LOD estimates. Especially the effects of solar radiation pressure on the right ascension of the ascending node are problematic.

2 Methods

Four different GNSS LOD data sets from Horozovic and Weber (2018) are used for estimating dUT1. Two of which consist solely of GPS data and two are calculated based on a combination of GPS and Galileo. For both of these variants one and three day solutions with one hour resolution are used. All of the data sets encompass the time frame from 1-JUL-2017 to 1-NOV-2017, use the ECOM model and are referred to as 1D GPS, 3D GPS, 1D GPSGAL and 3D GPSGAL in this paper. The data was processed with Bernese 5.2 (Dach et al., 2015) using data from over 190 IGS stations. For both the calibration of the offset in the GNSS LOD data, as well as the integration constant the IERS 14C04 data set (Bizouard et al., 2018), hereafter referred to as C04, is used. Furthermore, it is also used for the assessment of the estimated dUT1 values.

For the integration of the discrete LOD values the cumtrapz function in Matlab (R2016b) is used, which approximates the integral with trapezoids. A better approximation with integrating a cubic spline interpolation yields no significant improvements. In order to as-

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sess the quality of the dUT1 estimates, the LOD values are integrated over a specific integration period and the differences to the C04 dUT1 is calculated. Subsequently the integration window is shifted by one day and the process is repeated as long as there is data available. Thereafter the computed differences to C04 are stacked as a function of t-t_0 and the mean error as well as the RMSE are calculated. In Figure 1 an example of this stacking can be seen. Each of the faint lines is the difference between the estimated dUT1 for a specific integration period and the C04 dUT1. Due to the shifting by one day, each LOD value contributes to multiple integration periods and as a consequence some repeating patterns in the difference to C04 can be seen.

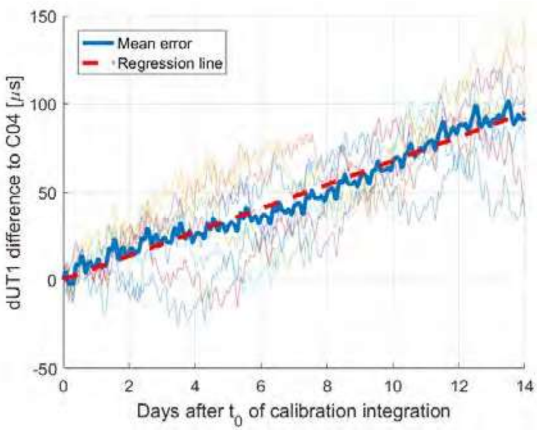

Fig. 1: Illustration of the stacking of integration periods and the bias estimation method.

The GNSS LOD data possesses an offset as is normal with this kind of data. Since dUT1 has to be known at t_0 it is sensible to assume that LOD is also known at that time. Therefore the bias can be estimated as the difference between the GNSS LOD and that known LOD at t_0. However, the calculated LOD difference at t_0 does not represent the offset over the integration period due to the noisy nature of the LOD data. Applying the same technique to smoothed GNSS LOD does not deliver satisfying results either.

In order to overcome this problem, a period with certain length prior to the integration start t_0 is chosen as a calibration period. During this period the LOD is integrated in a moving integration window and the resulting difference to C04 gets stacked as described above. Because the offset in the LOD data is approximately constant during an integration period, the calculated mean error of the dUT1 estimate drifts linearly. Thus the offset can be estimated by fitting a linear function to the mean error and taking the slope as is illustrated in Figure 1.

For comparison, the IGS corrects for the LOD bias by averaging the differences of the daily LOD values from every analysis center of the past weeks from the actual LOD value determined by the IERS Rapid Service. This produces an analysis center specific LOD bias which is used in the estimation (Kammeyer, 2000).

3 Results

Different parameters for the used calibration method were tested in order to get the best dUT1 estimates. Among them were weighted averaging of the different integration periods when calculating the mean error and different lengths for both the overall calibration window and the integration period within that window. The rational behind a weighted average of the different integration periods is that more recent periods may be more indicative of the current LOD offset. However, it is found that the more equally the different periods are weighted the better the calibration performs and a normal averaging yields the best results. It is important to note that the used calibration method introduces a weighting of the LOD values by itself. This is because the integration window is always completely contained in the calibration period and gets shifted by one day each time. As a consequence the first and the last LOD value contribute to one integration period, the second and second last to two and so on.

For the calibration length 7, 14, 21, 28 and 35 day periods were tested. Since the used calibration method acts similar to a moving mean, the resulting LOD bias signal gets smoother with a prolonged calibration period as can be seen in Figure 2. In order to achieve a good end result in terms of the RMSE as well as to preserve some of the bias variation a compromise of 21 days was chosen.

Concerning the length of the integration window within the calibration period 3, 5, 7, 10 and 14 days were tested, with 14 days resulting in the best calibration. For the following results a calibration with normal averaging, 21 day calibration period and 14 day integration window was used.

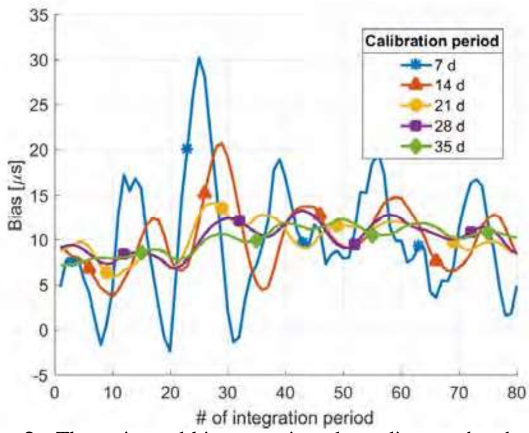

Fig. 2: The estimated bias over time depending on the chosen length of the calibration period.

Using this configuration good dUT1 estimates can be achieved. Figure 3 depicts the mean error and the RMSE of the dUT1 estimates when compared to the C04 as is described above. The calibration of the LOD bias was successful to a certain degree, as the mean errors of all four data sets are approximately zero. The three day solutions outperform the one day solutions by a large margin. The addition of Galileo to GPS for 1D yields a much bigger improvement than for 3D, for which the improvement is almost negligible. The RMSE for the best data set, 3D GPSGAL, after seven days is approximately 40 μs.

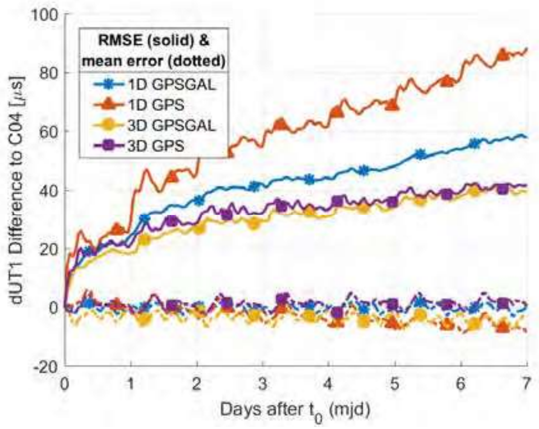

Fig. 3: The mean error and RMSE for the four data sets as a function of t-t_0.

Part of the reason that the calibration fails for the 1D GPS dataset is that the offset is assumed to be constant over the integration period. However, this assumption particularly does not hold true for this data set. Figure 4 shows the estimated biases for the different GNSS data sets over time. The bias for 1D GPS has the biggest amplitude and very steep slopes, which makes it hard to estimate with the employed method.

Both the mean error and RMSE of all solutions exhibit some periodic behavior. This is partly due to the fact that the C04 dUT1 values have a temporal resolution of one day while the GNSS LOD data has a temporal resolution of one hour. In order to compare the estimated dUT1 with the C04 data the latter is linearly interpolated. In addition, the LOD data contains some residual components of the ocean tides which were not removed by the model used in Bernese 5.2 (Dach et al., 2015).

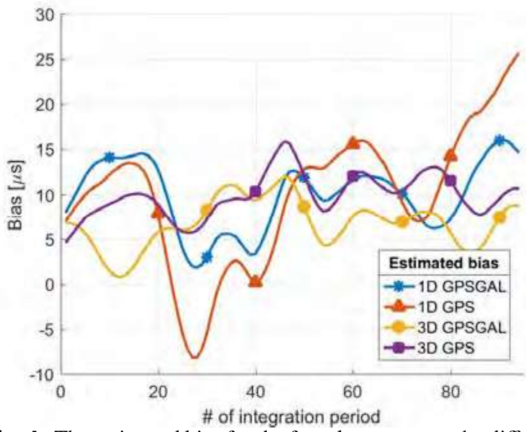

Fig. 4: The estimated bias for the four data sets over the different integration periods.

Assuming the distribution of the differences between GNSS LOD and C04 LOD is symmetrical and has an expectation value of zero, the RMSE follows a true random walk error and is proportional to the square root of time:

$$RMSE \propto \sqrt{t-t_0} \tag{2}$$

Figure 7 shows the RMSE of the dUT1 estimates over 7 days at the time-stamps of the C04 data. In addition the results of a curve fit of

$$RMSE \approx a * \sqrt{t-t_0} + d \tag{3}$$

to these points is shown. The estimated parameters as well as the goodness of fit (gof) values are listed in Table 1. The offset parameter d would be zero in an ideal case and the exhibited deviations from that theoretical value are due to remaining biases in the corrected GNSS LOD. These most likely stem from orbital perturbations, mainly due to radiation pressure, which were not completely modeled by the used ECOM model in Bernese 5.2 (Dach et al., 2015).

3D GPS and 3D GPSGAL follow the fitted function quite well although their *gof* parameters are not particularly good. On the other hand 1D GPS has the best *gof* parameters but exhibits a positive curvature after day 3 which means that the RMSE does not truly follow a square root function. This is explored further in the next section with a 14 day integration period.

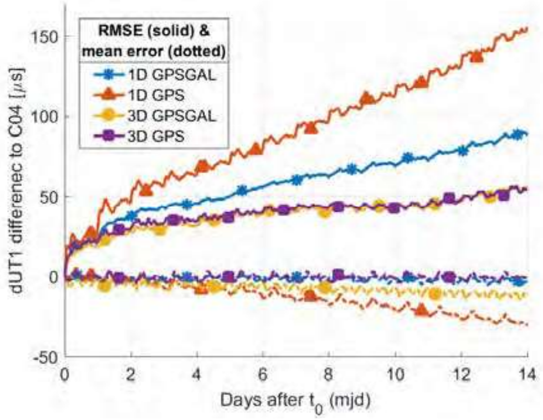

Fig. 6: The mean error and RMSE for the four data sets as a function of t-t_0 for a longer integration window of 14 days.

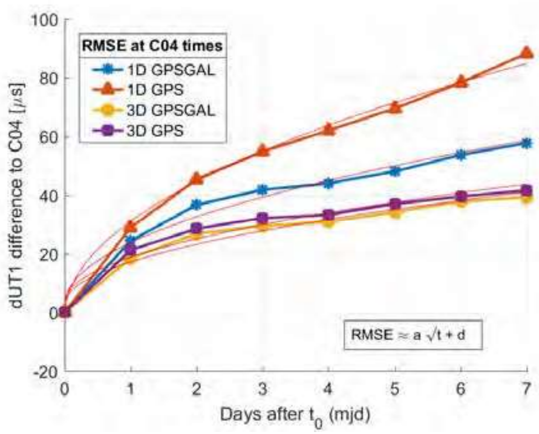

Fig. 5: Result of fitting a square root function with an offset term to the RMSE at the C04 sampling points.

Table 1: Fitting parameters and goodness of fit for 7 days

Data set	a	d	R^2	RMS
GPS 1D	32.71	-1.70	0.995	2.120
GPSGAL 1D	21.22	2.56	0.985	2.466
GPS 3D	15.20	3.56	0.966	2.662
GPSGAL 3D	14.51	2.68	0.974	2.220

Figure 6 shows the progression of the mean error and RMSE for an integration period of 14 days. The 1D GPS estimate is by far the worst and its RMSE increases linearly with time instead of the square root of time. This means that the conditions of a pure random walk error are not met, which is apparent when looking at the trend of the respective mean error which is quite significant. The already small advantage of 3D GPSGAL over 3D GPS pretty much vanishes after approximately 9 days. However, when looking at the trend of their respective mean errors, it becomes clear that 3D GPS profits of a much better bias estimation than 3D GPSGAL. Since their RMSE are so similar it can be inferred that GPSGAL 3D has a lower scattering but the bias is either not as constant as for the other data sets, harder to be estimated or both. It is interesting to note that bias estimation for GPS improves when going from the 1D to the 3D solution but for GPSGAL it is the other way around. The reasons for this have to be investigated.

Figure 7 depicts the RMSE of the dUT1 estimates over 14 days at the sampling points of the C04 data set and the results of a curve fit according to Equation 3. The parameters resulting from this fit are listed alongside the *gof* measurements in Table 2. In addition the figure also shows a RMSE reference, which was taken from a paper about estimation of a dUT1-like quantitiy based on GPS orbit planes (Kammeyer, 2000). Within that paper the RMS change over time was investigated over longer periods and the resulting RMS approximation is given with 30μ s $\sqrt{w}$ wherein w is the elapsed time in weeks.

Table 2: Fitting parameters and goodness of fit for 14 days

Data set	a	d	R^2	RMS
GPS 1D	40.58	-12.04	0.972	7.590
GPSGAL 1D	21.61	2.70	0.989	2.516
GPS 3D	12.33	7.77	0.943	3.346
GPSGAL 3D	12.70	5.30	0.957	2.835

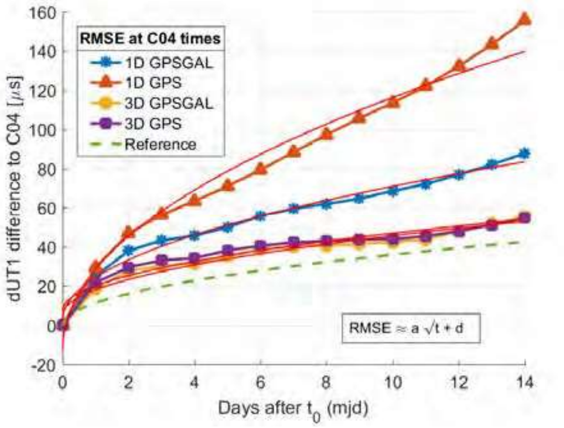

Fig. 7: RMSE over 14 days at C04 times and the curve fitting results. Additionally a reference curve is shown.

Interestingly the estimate for the a parameter decreases quite drastically between the 7 and 14 day fits for the two 3D solutions. The d parameter on the other hand gets bigger. This points to disturbances of the pure random walk error that get absorbed in the a parameter in case of the 7 day fit, but move to the d term in the 14 day fit. The 1D GPS dUT1 estimate performs even worse, has bad *gof* parameters and exhibits a positive curvature that is even more visible in Figure 7 than it was in the previous ones.

Both 3D solutions perform similar, although a bit worse than the reference data. However, this is due to the sharp increase at the beginning of the integration period. The actual course of the RMSE as described by the a term is very similar with 12.33 and 12.70 compared to 11.34. The a term for the reference data results from a scaling of the weekly coefficient by $\frac{1}{\sqrt{7}}$. The reasons for the steep increase in the RMSE at the very beginning of the integration period need to be investigated further.

4 Conclusions

The presented bias estimation method yields good results and enables to obtain usable dUT1 estimates. It works best for the used 1D GPSGAL and 3D GPS data sets while struggling with GPS 1D. Both 3D data sets yield good dUT1 estimates with the 3D GPSGAL doing so despite a slightly worse bias estimation based on the mean error. 1D GPS results in the worst bias estimation and the worst RMSE by a big margin. This is most likely due to a higher variability of the bias during a single integration period.

The best case RMSE after 7 days with the used data is 40 μs with the 3D GPSGAL data set. After 14 days the estimates based on 3D GPSGAL and 3D GPS are nearly identical and exhibit an RMSE of almost 60 μs. All those accuracy measures do not include the uncertainty of the initial dUT1 value at t_0. The attainable accuracy of dUT1 from VLBI intensive sessions is 20 μs for the best global intensives (Schuh and Böhm, 2013) and about 40 μs for European intensives (Schartner et al., 2018).

When comparing the obtained dUT1 RMSE to the aforementioned reference data set the former falls behind a bit. While the best GNSS data sets, 3D GPS and 3D GPSGAL exhibit a similar a parameter for the curve fit, the steep incline of the RMSE at the beginning and the accompanying high d value lead to a worse overall performance.

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