ESTIMATION OF NUTATION RATES FROM COMBINATION OF RING LASER AND VLBI DATA

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ABSTRACT. Ring laser gyroscopes (RLG) are instruments measuring inertial rotations locally and in real-time without the need for an external reference system. They are sensitive to variations in the instantaneous rotation vector, therefore they are considered as a potential complement to space geodetic techniques for studying Earth rotation. In this work we examine the usability of ring laser observations for estimation of nutation rates. We investigate possibilities of computing those parameters from only one ring laser and we simulate the usage of several instruments. We also combine simulated RLG observations with actual Very Long Baseline Interferometry VLBI data and compare them with real Wettzell RLG data. Our results attest to the theoretical possibility of estimating nutation rates, albeit with a number of restrictive assumptions.

1. MOTIVATION AND ASSUMPTIONS

Ring laser gyroscopes are instruments which present a dynamical approach to the determination of Earth rotation parameters. They enable measuring Earth rotation on the surface of the Earth, as they are sensitive to variations in the instantaneous rotation vector. They are considered as a potential complement to space geodetic techniques in studying Earth rotation. To date many experiments have been conducted in order to investigate possible advantages of combining ring laser observations and data from space geodetic techniques, especially from Very Long Baseline Interferometry. The majority of those experiments concern polar motion and universal time (UT1) variations. In this work we examine the potential usage of ring laser observations for estimating nutation rates simulating two weeks of ring laser observations and checking the conditions under which such an estimation is possible. Our work is divided into three parts. The first part concerns the determination of nutation rates from ring laser data only, in the second part we address a combined solution with VLBI data, and in the last part we compare our results obtained from simulated RLG observations with those from real Wettzell data.

The conducted simulation is based on the relative Sagnac frequency equation (Nilsson et al., 2012):

$$\Delta S = \cot\varphi(m_x \cos\lambda + m_y \sin\lambda) + m_z + \Delta S_{\mathrm{tilt}} + \Delta S_{\mathrm{instr}}, \tag{1}$$

where φ and λ are latitude and longitude of the instrument, ΔS_{tilt}, $\Delta S_{\mathrm{instr}}$ are errors caused by tilts and instrumental imperfections, and m_x, m_y, m_z are dimensionless parameters defining perturbations of the instantaneous rotation vector $\vec{\omega} = \Omega_0[m_x\ m_y\ 1 + m_z]^T$ (with Ω_0 denoting the mean angular speed of rotation). Hence m_x, m_y describe polar motion of the Instantaneous Rotation Pole (IRP), which relates to the motion of the Celestial Intermediate Pole (CIP) via (Cerveira et al., 2009):

$$m_x = \frac{1}{\Omega_0}(\Omega_0 x_p - \dot{y}_p + (\dot{X} + d\dot{X})\sin\theta - (\dot{Y} + d\dot{Y})\cos\theta), \tag{2}$$

$$m_y = \frac{1}{\Omega_0}(-\Omega_0 y_p - \dot{x}_p + (\dot{X} + d\dot{X})\cos\theta + (\dot{Y} + d\dot{Y})\sin\theta), \tag{3}$$

where θ denotes the Earth rotation angle, x_p and $-y_p$ are terrestrial components of the CIP, describing polar motion (PM), X and Y are coordinates of the CIP in Geocentric Celestial Reference System (GCRS),

dX, dY are the celestial pole offsets describing the celestial perturbation of the CIP, that is nutation, and the dot represents the time derivative. Variations of the axial component of the instantaneous rotation vector are expressed by (Nilsson et al., 2012):

$$m_z = 1.0027 d\dot{UT}1, \tag{4}$$

where dUT1 = UT1 − UTC (with UTC denoting the coordinated universal time) and 1.0027 is the proportionality factor between the solar and the sidereal time scales.

The design matrix A is constructed assuming one value of the nutation offset rates $d\dot{X}$, $d\dot{Y}$ and one value of the instrumental error per day as unknowns. The right-hand side vector is $L = \Delta S_{\text{obs}} - \Delta S_{\text{comp}}$ + random noise. For ΔS_{obs} we use values obtained with PM and dUT1 from the C04 08 EOP series, taking into account ocean tides, and with nutation from the IAU 2006/2000 model (X, Y) and offsets (dX, dY) from the C04 08 series. ΔS_{comp} is derived the same way, with the exception of excluding nutation offsets from the C04 08 series. The random noise is generated by a random multiplication with a prescribed accuracy level. The weighting matrix P contains σ^{-2} on the diagonal with $\sigma = 10^{-11}$, since it turned out that the accuracy level 10^{-8} suggested in (Nilsson et al., 2012) is not adequate to our task. To obtain satisfactory results we had to increase the accuracy level by three orders of magnitude. The first day of observations is assumed to be September 15, 2011, as it is the first day of a two-week campaign of continuous VLBI sessions CONT11.

The first question we are looking for an answer is how many instruments do we need. For this end we estimate nutation rates assuming the use of one to six instruments, located regularly over the world. The locations of RLG are assumed in the same places as the existing VLBI stations: Wettzell, Hobart, Fortaleza, Kokee, Badary and Syowa. First we assume one RLG in Wettzell, then two (Wettzell and Hobart), then three (Wettzell, Hobart and Fortaleza), and so on. The second problem we want to investigate is the potential advantage of a combination of RLG and VLBI data. For this purpose we process CONT11 sessions using the VieVS software (Böhm et al., 2012) following (Nilsson et al., 2012), and then combine simulated RLG data with VLBI observations on the normal equation level. As there are no common parameters for both techniques the new normal equation matrix and the right-hand side vector are prepared as follows:

$$N_{\text{new}} = \begin{bmatrix} N_{\text{VLBI}} & 0 \\ 0 & N_{\text{RLG}} \end{bmatrix} \text{ and } L_{new} = \begin{bmatrix} L_{\text{VLBI}} \\ L_{\text{RLG}} \end{bmatrix}, \tag{5}$$

and taking into account constraints, i.e. nutation rates = nutation finite differences

$$N_C = \begin{bmatrix} N_{\text{new}} & C^T \\ C & 0 \end{bmatrix} \text{ and } L_C = \begin{bmatrix} L_{\text{new}} \\ 0 \end{bmatrix}. \tag{6}$$

For our combination we use simulations based on one, three and six instruments, starting with the present-day accuracy level of 10^{-8} and followed by refinement to 10^{-9}, 10^{-10} and 10^{-11}. The last part of the procedure consists in a comparison of our simulated RLG observations with real data, from the Geodetic Observatory Wettzell. At this stage we simulate observations for one instrument only located at Wettzell, assuming the present-day accuracy level of 10^{-8} and a time span of 24 days (May 1–25, 2010).

2. RESULTS

Results of the first part, i.e. dX and dY rates derived under the assumptions of using of one to six RLG, are shown in Fig. 1, and the corresponding RMS errors of residuals are summarized in Table 1.

The graphical results are compared with a model time series taken to be nutation rates as computed based on the C04 08 series. It can be seen that for one and two devices the results are less consistent with the model as those obtained from three and more RLG. This observation conforms with the results

	1	2	3	4	5	6
$d\dot{X}$	0.5407	0.2480	0.0601	0.0555	0.0466	0.0450
$d\dot{Y}$	0.4634	0.3654	0.1237	0.1195	0.1373	0.1243

Table 1: RMSs of the residuals with respect to the number of used ring lasers. Values in 10^{-7}mas/s.

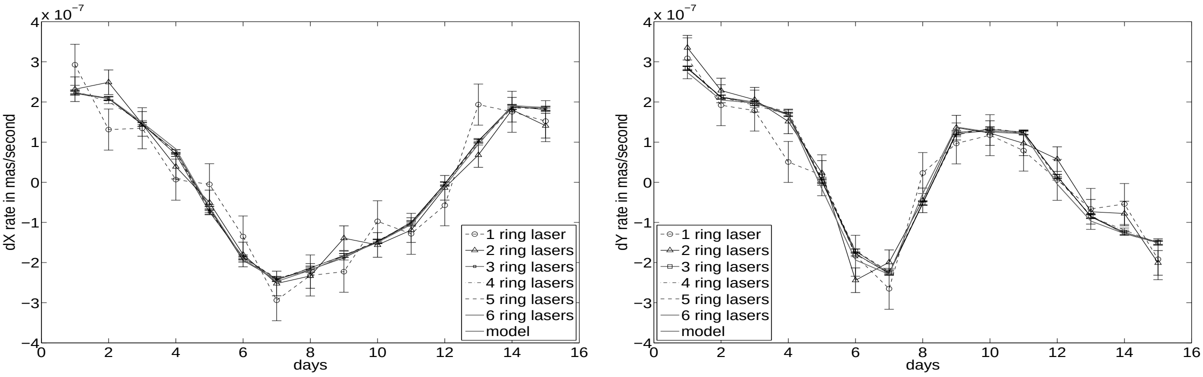

Figure 1: Nutation rates in X and Y, from RLG observations depending on the number of instruments.

of Nilsson et al. (2012), who concluded that for estimation of the complete Earth rotation vector at least three ring lasers with different orientations would be needed.

Results for the combined solution are shown in Fig. 2. It turns out again, that the present-day accuracy level of RLGs is not sufficient for our task. For a level of 10^{-8} there is no difference if we have one or more instruments, the combination does not provide a satisfactory solution. Similarities with the expected model curve emerge with $\sigma = 10^{-9}$ and are further enhanced for the case of refined accuracy level. As apparent from Figs. 2a and 2b the differences between values obtained from one RLG and from three and six are not so large if we use 10^{-9}, but when assuming 10^{-10} the differences are considerable (Figs. 2c and 2d). This might again suggest that one instrument is not enough, even for a combined solution, but it also shows that the present-day accuracy level is not adequate for the task.

After analyzing simulated data, we estimate nutation rates based on real RLG data from the Wettzell instrument. We use the same algorithm, but for ΔS_{obs} we take real observations, after accounting for

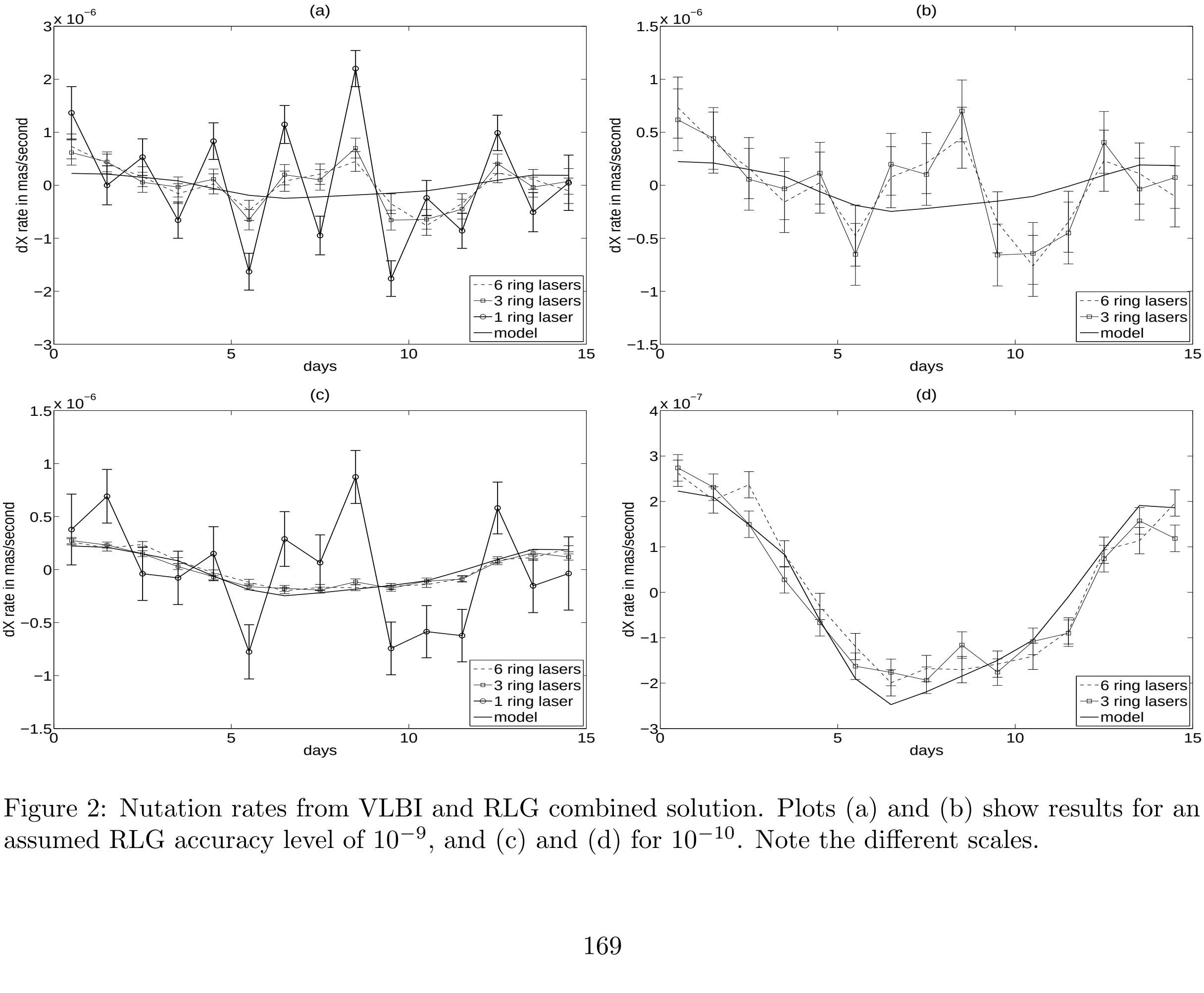

Figure 2: Nutation rates from VLBI and RLG combined solution. Plots (a) and (b) show results for an assumed RLG accuracy level of 10^{-9}, and (c) and (d) for 10^{-10}. Note the different scales.

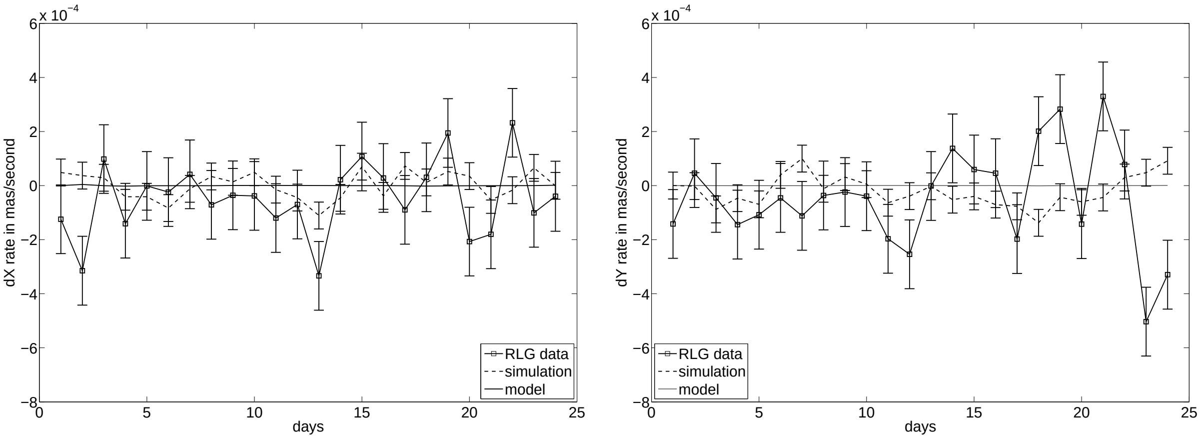

Figure 3: Comparison of results from simulated and real Wettzell RLG data. The solid line with square markers shows results obtained based on real data and the dashed one – based on simulated observations.

tilt corrections. Figure 3 shows that the results from simulated observation, though not satisfactory, nevertheless better agree with the model than the results based on real data. This might indicate that in our investigation we assume a too simple model for the Sagnac frequency or that we did not take into account some important local effects.

3. CONCLUSIONS

We demonstrated that the estimation of nutation rates from ring laser observations is possible. A comparison of our simulations with real ring laser data showed that the recent accuracy is not sufficient to detect nutation rates in the time series. In order to improve VLBI measurements of nutation rates at least three RLG instruments and an accuracy level three orders of magnitude higher than nowadays are required. This is hard to achieve as shown by Schreiber et al. (2011). Moreover it should be mentioned that we used a very simple model of relative Sagnac frequency and did not take into account any local effects and we considered only the geometrical aspect associated with the localization of instruments. This discouraging conclusion is not surprising, as the ring laser is not very sensitive to nutations, and it was never considered to measure nutation rates at all. In this context it should also be stressed that one instrument measuring one component is compared with a global set of radio telescopes. However, the ring laser yields valuable Earth rotation data in the terrestrial frame, as it is the only instrument measuring polar motion of the rotation axis without the need of conventions. Recent improvements in the long term stability of the ring laser are promising that investigations of this topic should be continued.

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